Code No. : 5442/N

# FACULTY OF INFORMATICS

## B.E. 2/4 (IT) II Semester (Main) Examination, May/June 2012

## Probability and Random Process

Time : 3 Hours]

[Max. Marks : 75

***Note :*** *Answer **all** questions from Part **A**. Answer **any five** questions from Part **B**.*

### PART – A

(25 Marks)

1. What is the difference between total independence and mutual independence. 2
2. Each of the two persons A and B tosses 3 fair coins. What is the probability that they obtain the same number of heads ? 2
3. Define probability density function and state its properties. 2
4. Show that the area under the exponential distribution curve is one. 3
5. Find characteristic function of Poisson distribution. 3
6. Define independent random variable and prove that E(XY) = E(X) E(Y) if X and Y are independent Random variables. 3
7. Define Auto-Correlation function of a stationery process and state any two important properties. 3
8. Define Gaussian process. 2
9. Define white noise and filters. 2
10. Show that $2^n - (n+1)$ equations are needed to establish mutual independence of 'n' events. 3

### PART – B

(50 Marks)

11. a) State and prove addition theorem for 'n' events. 5

    b) Suppose box-1 contains a white balls and b black balls, and box-2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball ? 5

12. a) Train X and Y arrive at a station at random between 8.00 a.m. and 8.20 a.m. Train X stops for four minutes and train Y stops for five minutes. Assuming that the train arrive independently of each other, determine :

   i) Train X arrives before Train Y.

   ii) Two trains meet at the station.

   iii) If they met at the station, what is the probability that X arrived before Y ? 7

   b) Suppose the life length of an appliance has an exponential distribution with mean life 10 years. A used appliance is bought by someone. What is the probability that it will not fail in the next 5 years ? 3

13. a) A fair coin is tossed 10000 times. What is the probability that the number of heads is between 4900 and 5100 ? 5

   b) The probability of hitting and Aircraft is 0.001 for each shot. How many shots should be fired so that the probability of hitting with two or more shots is above 0.95 ? 5

14. a) If $Y = X^2$, where X is a Gaussian random variable with zero mean and variance $\sigma^2$, find the pdf of the random variable Y. 3

   b) If the density function of RV X is given by 7

$$f(X) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

   i) Find the value of a

   ii) Find the cdf of X

   iii) If $x_1$, $x_2$ and $x_3$ are independent observations of X, what is the probability that exactly one of these is greater than 1.5 ?

15. a) If the joint pdf of (X, Y) is $f(x, y) = 6\,e^{-2x-3y}$, $x \ge 0$, $y \ge 0$, find the marginal density of X and conditional density of Y given X. 5

b) The following table represents the joint probability distribution of the discrete RV (X, Y). Find all the marginal and conditional distributions. 5

| Y | X | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | 1/12 | 1/6 | 0 |
| 2 | 0 | 1/9 | 1/5 |
| 3 | 1/18 | 1/4 | 2/15 |

16. a) Two random processes X(t) and Y(t) are defined by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t$ and $Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$. Show that X(t) and Y(t) are jointly wide sense stationary, if A and B are uncorrelated RVs with zero means and the same variances and $\omega_0$ is a constant. 5

b) Define the semi-random telegraph signal process and random telegraph signal process and prove also that the former is evolutionary and the latter is wide sense stationary. 5

17. a) It is given that $R_x(\tau) = e^{-|\tau|}$ for a certain stationary Gaussian random process {X(t)}. Find the joint pdf of the RVs X(t), X(t + 1), X (t + 2). 5

b) If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$, where A is a constant, $\theta$ is a random variable with a uniform distribution in $(-\pi, \pi)$ and {N(t)} is a band-limited Gaussian white noise with a power spectral density. 5

$$S_{NN}(\omega) = f(x) = \begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & \text{elsewhere} \end{cases}$$

Find the power spectral density of {Y(t)}. Assume that N(t) and $\theta$ are independent.