## - FACULTY OF INFORMATICS

## B.E. $2 / 4$ (IT) I Semester (Main) Examination, December 2010 DISCRETE MATHEMATICS

Time : 3 Hours]

Note: Answer all questions of Pa $\sum^{\text {LIBRARY }}$ A, Answe ${ }^{\circ}$, pe questions from Part - B.

PART - A
[Max. Marks : 75

1. Construct truth table for $[(p \vee q) \wedge(\sim r)] \leftrightarrow q$.
2. Write negation of
"There is an integer x such that x is even and x is prime" by changing quantifiers.
3. Check whether the following relation is a function. Define $R$ by $x R y$ if $\mathrm{x}^{2}+\mathrm{y}^{2}=1$ where $\mathrm{x}+\mathrm{y}$ are real numbers such that $\mathrm{o} \leq \mathrm{x} \leq 1,0 \leq \mathrm{y} \leq 1$.
4. Prove that for all integers $n \geq 4,3^{n}>n^{3}$ by induction.
5. The recursive definition of the binomial co-efficients.
$C\left(n_{1} k\right)$ is $C\left(n_{1} n\right)=1, C\left(n_{1} 0\right)=1$ and $C(n, k)=C(n-1, k)+C(n-1, k-1)$ if $\mathrm{n}>\mathrm{k}>\mathrm{o}$ then expand $\mathrm{C}(5,2)$ to express interms of quantifiers defined by the basics.
6. How many ways are there to roll 2 dice to yield asum that is divisible by 3 ?
7. If $\mathrm{V}=\left\{\mathrm{v}_{1} \mathrm{v}_{2} \ldots \ldots \ldots . . \mathrm{v}_{\mathrm{n}}\right\}$ is the vertex set of a non-directed graph then prove that $\sum_{i=1}^{n} \operatorname{deg}\left(v_{i}\right)=21 E l$.
8. Define
a) cycle graph
b) Bipartite graph $v_{3}$.
9. Define
a) POSET
b) Lattice
10. Find the incident matrix of (hemand:

PART - B
(50 Marks)
11. a) Check for tautology of the following :

$$
[(p \rightarrow q) \wedge(r \rightarrow s) \wedge(p \vee r)] \rightarrow(q \vee s)
$$

b) Discuss the various methods of proving statements.
12. a) Demonistrate that ' $S$ ' is a valid inference from the premises $\mathrm{p} \rightarrow \sim \mathrm{Q}, \mathrm{Q} \vee \mathrm{R}, \sim \mathrm{S} \rightarrow \mathrm{P}$ and $\sim \mathrm{R}$.
b) How many 4 - digit numbers can be formed with the ten - digits $\{0,1,2, . ., 9\}$ if i) repetitions are allowed ii) repetitions are not allowed.
13. a) Find a recurrence relation and give initial condition for the number of bit string of length ' $n$ ' that donot have two consecutive o's. How many such bit strings are there of length five ?
b) Solve $a_{n}-4 a_{n-1}+4 a_{n-2}=3 n+2^{n}$ with $a_{0}=1, a_{1}=1$.
14. a) Draw the Hasse diagram for divisibility on the set $\{1,2,3,5,7,11,13\}$. Is it a Lattice?
b) Define Euler circuit and Euler path. Find which of the following graph have Euler circuit or Euler path or not having both.

$G_{1}$

$G_{2}$

${ }_{9}$
15. a) State and prove five - color theorem.
b) State and prove Euler's formula for connected planar simple graph.
16. a) Use DFS algorithm to find spanning tree for the following graph.

b) Describe Kruskal's algorithm for a minimal spanning tree with an example.
17. a) Prove that a tree with $n$-vertices has exactly ' $n-1$ ' edges.
b) Show that $K_{n}$ is planar for $1 \leq n \leq 4$.

