# FACULTY OF INFORMATICS <br> B.E. $2 / 4$ (IT) II Semester (Main) Examination, June 2010 PROBABILITY AND RANDOM PROCESS 

Time: 3 Hours]
[Max. Marks: 75
Instructions : Answer all questions from Part A.
Answer any five questions from Part $\boldsymbol{B}$.
PART - A

1. State addition theorem for n events.
2. A player tosses four coins at a time, find the probabilities at leasttwo heads and at least two tails.
3. Find the characteristic function of p.m.f.

$$
p(x)=\frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \quad \begin{array}{ll}
\lambda>0  \tag{2}\\
x \geq 0
\end{array}
$$

4. State Wineer Kichneey's relation. ..... 2
5. Define Random vector, give one example. ..... 2
6. Derive variance for normal (Gaussion) random variable. ..... 3
7. If $X$ and $Y$ are r.v.'s $a, b$ are constants prove that $\mathrm{V}(\mathrm{ax}-\mathrm{bY})=\mathrm{a}^{2} \mathrm{~V}(\mathrm{X})+\mathrm{b}^{2} \mathrm{~V}(\mathrm{Y})-2 \mathrm{abCOV}(\mathrm{X}, \mathrm{Y})$. ..... 3
8. Check the following are suitable autocorrelation functions
i) $\mathrm{A} \cos \omega t$
ii) $\mathrm{A} \sin \omega t$ ..... 3
9. Define white noise. ..... 2
10. State the properties of spectral density function. ..... 3
11. a) State and prove Baye's theorem.
b) A letter is known have come from either TATANAGAR or CALCUTTA, on the envelope the just two consecutive letters TA are variable. Find the probability that the letter has come from CALCUTTA.
12. a) Find mean and variance for the following function

$$
f(x)=\left\{\begin{array}{lll}
K x e^{-x} & n>0 ; & K \text { constant }  \tag{5}\\
0 & \text { otherwise }
\end{array}\right.
$$

b) State and prove additive property of gamma variables.
13. a) State the properties auto correlation.
b) If $X(t)$ is random telegraph signal process with $E(X(t))=0$ and $R(T)=e^{-2 \lambda / T}$ find mean and variance of the time average of $\{x(t)\}$ over $(-T, T)$, is it mean ergodic.
14. If $U(t)=X \cos t+Y \sin t$ and $V(t)=Y \cos t+X \sin t$ where $X$ and $Y$ are independent r.v.'s such that $\mathrm{E}(\mathrm{X})=0=\mathrm{E}(\mathrm{Y}) ; \mathrm{E}\left(\mathrm{X}^{2}\right)=\mathrm{E}\left(\mathrm{Y}^{2}\right)=1$; Show that $\{\mathrm{U}(\mathrm{t})\}$ and $\{\mathrm{V}(\mathrm{t})\}$ are individually stationary in the wide sense (WSS) but they are not jointly W.S.S. $\mathbf{1 0}$
15. a) Find the power spectral density of the random binary transmission process whose auto correlation function is

$$
R(t)= \begin{cases}1-\frac{|t|}{T} & |t| \leq T  \tag{5}\\ 0 & \text { elsewhere }\end{cases}
$$

b) State and prove Tchebycheff's inequality. 5
16. The joint random variables $(\mathrm{X}, \mathrm{Y})$ probabilities function is given by

| X | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $1 / 32$ | $2 / 32$ | $2 / 32$ | $3 / 32$ |
| 1 | $1 / 16$ | $1 / 16$ | $1 / 8$ | $1 / 8$ | $1 / 8$ | $1 / 8$ |
| 2 | $1 / 32$ | $1 / 32$ | $1 / 64$ | $1 / 64$ | 0 | $2 / 64$ |

find :
i) $\mathbf{P}(\mathrm{X} \leq 1)$
ii) $\mathrm{P}(\mathrm{X} \leq 1, \mathrm{Y} \leq 3)$
iii) $\mathrm{P}(\mathrm{x} \leq 1, \mid \mathrm{Y} \leq 3)$
iv) $\mathrm{P}(\mathrm{X}+\mathrm{Y} \leq 4)$
v) $\mathrm{P}(\mathrm{Y} \leq 3 / \mathrm{X} \leq 1)$
17. A white noise of Gaussion process in zero, and $S(w)=\frac{N_{o}}{2}$ applied to a low pass R.C. filter whose transfer function $H(f)=1 / 1+i 2 \pi f R_{c}$.

