## FACULTY OF ENGINEERING

## B.E. 2/4 (Main) Examination, June 2010 MATHEMATICS - IV

Time: 3 Hours]
Instructions : Answer all questions of Part A. Answer five questions from Part $\boldsymbol{B}$.
PART - A
 where C is the circle $|z|=2$.
a) $-2 \pi i$
b) $2 \pi \mathrm{i}$
c) $4 \pi \mathrm{i}$
d) None
2. Write the Cauchy-Riemann equations in polar form.
3. Find the image of the curve (the circle) $|z-3|=5$ under the mapping $w=\frac{1}{z}$.
4. Expand the function $\log (1+z)$ as a Taylor's series about $z=0$.
5. A random variable $x$ has the probability density function $f(x)=6 x(1-x) \quad 0 \leq x \leq 1$.
Find mean, median and mode.
6. Indicate whether the following statement are true or false.
a) If $f(z)=e^{1 / z}$; the singularity at $z=0$ is called removable singularity.
b) $E(a x+b)=a^{2} E(x)+b$
c) Arithmetic mean of regression coefficients is greater than the correlation coefficient.
7. Write down conditions for applying $\chi^{2}$ test.
8. The first four moments of $x$ about $x=5$ are $1,-5,15$ and 30 . Find the corresponding four moments about the mean.
9. The normal distribution is a limiting form of binomial distribution if
a) $\mathrm{n} \rightarrow \infty, \mathrm{p} \rightarrow 0$,
b) $\mathrm{n} \rightarrow 0, \mathrm{p} \rightarrow \mathrm{q}$,
c) $\mathrm{n} \rightarrow \infty, \mathrm{p} \rightarrow \mathrm{n}$
d) $\mathrm{n} \rightarrow \infty$ and neither p nor q is small.
10. A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.
PART - B
(50 Marks)
11. a) Determine the analytic function whose real part is $e^{2 x}(x \cos 2 y-y \sin 2 y)$.
b) If $f(z)$ is a regular function of $z$, prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)|f(z)|^{2}=4\left|f^{\prime}(z)\right|^{2}$.
12. a) If $f(z)$ is analytic and $f^{\prime}(z) \neq 0$ in the region $R$ of the $z$-plane, then show that the mapping $w=f(z)$ is conformal at all the points of $R$.
b) State and prove Cauchy's integral theorem.
13. State the residue theorem, and evaluate
i) $\oint_{C} \frac{z-3}{z^{2}+2 z+5} d z$, where $C$ is the circle $|z|=1$
ii) $\oint_{C} \frac{z}{(z-1)(z-2)^{2}} d z$, where $C$ is the circle $|z-2|=\frac{1}{2}$.
14. a) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?
b) Two unbiased dice are thrown. Find the expected values of the sum of number of points on them.
15. a) Fit a Poisson distribution to the following :

| $x:$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{f}:$ | 192 | 100 | 24 | 3 | 1 |

b) Find the m.g.f. and mean for Chi-square function.
16. Obtain the coefficient of correlation for the following data :

| $\mathbf{X}$ | 68 | 64 | 75 | 50 | 64 | 80 | 75 | 40 | 55 | 64 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{Y}$ | 62 | 58 | 68 | 45 | 81 | 60 | 68 | 48 | 50 | 70 |

17 a) Two random variables have the regression lines $3 x+2 y=26$ and $6 x+y=31$.
Find the mean value and the correlation coefficient between $x$ and $y$.
b) Two random samples drawn from two normal population have the variable values as below.

| Sample I | 19 | 17 | 16 | 28 | 22 | 23 | 19 | 24 | 26 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample II | 28 | 32 | 40 | 37 | 30 | 35 | 40 | 28 | 41 | 45 | 30 | 36 |

Obtain the estimate of the variance of the population and test whether the two population have the same variance.

