Code No.: **5003/M**

FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (Common to all branches) Examination, May/June 2012

MATHEMATICS – II

Time : 3 Hours] [Max. Marks : 75

Answer **all** *questions from Part-A*
Answer any **five** *questions from Part-B.*

Part A — (Marks : 25)

1. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. 3

2. I.F of $xy(1+xy^2)\frac{dy}{dx} = 1$ is ________. 2

3. Solve $(D^4 + D^2 + 1)\, y = 0$. 2

4. Solve $(D^2 + 9)\, y = \sin 3x$. 3

5. Find the value of $P^1_{n(-x)}$. 2

6. Show that $J_{\frac{-1}{2}}(x) = \sqrt{2/\pi x}\cdot\cos x$. 3

7. Prove that $T_{n+1}(x) = 2x\, T_n(x) - T_{n-1}(x)$. 2

8. Show that $\int_0^1 \log\left(\frac{1}{y}\right)^{n-1} dy = \Gamma n$. 3

9. Find the Laplace transform of $\sin^3 2t$. 3

10. Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$. 2

Part B – (Marks: 50)

11. (a) Solve $x\frac{dy}{dx} + y = x^3 y^6$. 5

(b) Show that the differential equation for the current i is an electrical circuit containing an inductance L and a resistance R in series and acted on by an electromotive force $E \sin Wt$ satisfies the equation $L\frac{di}{dt} + Ri = E \sin wt$. 5

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12. (a) Solve $(D^2 - 3D + 2)\, y = x\, e^{3x} + \sin 2x$. 5

(b) Using the method of variation of parameters, solve $y'' - 2y' + y = e^x \log x$. 5

13. Solve by series solution method of the equation

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0 \text{ , about } x = 0.$$ 10

14. (a) Prove $(2n + 1)x\, P_n(x) = (n + 1)\, P_{n+1}(x) + n\, P_{n-1}(x)$. 5

(b) Show that $(1-2xz + z^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x).z^n$. 5

15. (a) State and prove orthogonal properties of chebyshev polynomials of the first kind. 5

(b) Prove that $[T_n'(x)]^2 - T_{n+1}(x) T_{n-1}(x) = 1-x^2$. 5

16. (a) Evaluate $\int_0^1 \frac{dx}{(1-x^n)^{1/n}}$. 5

(b) Prove that $J_2^1(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$. 5

17. (a) Apply convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. 5

(b) Apply the method of Laplace transform to solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$. 5