FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (New) (Common to all Branches) (Main) Examination, June 2011

MATHEMATICS – II

Time : 3 Hours] **[Max. Marks : 75**

Note : Answer **all** questions from Part – **A**. Answer any **five** Questions from Part – **B.**

PART – A **(Marks : 25)**

1. Eliminate the arbitrary constants from
$y = a \cdot e^{x} + b \cdot e^{2x}$
and form differential equation. 2

2. Solve, $(3x^2 + 2e^y)dx + (2xe^y + 3y^2)dy = 0$. 3

3. Show that the set of function $\left\{x, \frac{1}{x}\right\}$ from series of the equation $x^2y'' + xy' - y = 0$ 3

4. Solve $y'' - y = 0, y(0) = 0, y'(0) = 2$. 2

5. Define singular and regular singular points. 2

6. Show that $P_2(u) = \frac{1}{2}(3u^2 - 1)$ 3

7. Find the value of $\Gamma\left(\frac{7}{2}\right)$. 2

8. Find the solution of the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{16}\right)y = 0$ in terms of Bessel's function. 3

9. Find Laplace transform of t sinh t. 2

10. Find inverse Laplace transform of
$\frac{s+2}{s^2 - 4s + 3}$ 3

PART – B **(Marks : 5 × 10 = 50)**

11. (a) Find the integrating factor and hence solve the differential equation $(x^2 + y^2)\,dx - 2xy\,dy = 0$ 5

(b) Show that the family of curves $\frac{x^2}{c} + \frac{y^2}{c+2} + 1 = 0$, is self orthogonal. 5

12. (a) Find the general and the singular solution of Clairaut's equation $y = xy' - (y')^3$. 5

(b) Solve the critical value problem 5
$y''' + 3y'' - 4y = 0,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = 1/2$.

13. (a) Find the general solution of $y'' + 3y' + 2y = 2e^x$. 5

(b) If $y_1 = e^{-2x}$ is the one of the solutions of $y'' - y' - 6y = 0$, find other solution by reducing the order of the differential equation. 5

14. Find the series solution about $x = 0$, of the differential equation $x(1 + x)y'' + 3xy' + y = 0$. 10

15. (a) Prove that :
$(n + 1)p_{n+1}(x) = (2n + 1)\, x\, p_n(x) - n\, p_{n-1}(x)$. 5

(b) Prove that $\beta(m, \tfrac{1}{2}) = 2^{2m-1}\, \beta(m, n)$. 5

16. (a) Express the integral $\int_0^1 \frac{dx}{\sqrt{1 - x^4}}$ in terms of Gamma functions. 5

(b) Prove that 5

$$\frac{d}{dx}\left[J_n^2(x)\right] = \frac{x}{2n}\left[J_{n-1}^2(x) - J_{n+1}^2(x)\right]$$

17. (a) Using convolution theorem, evaluate 5

$$L^{-1}\left(\frac{1}{(s + 1)(s + 9)^2}\right)$$

(b) Solve, $\frac{d^2y}{dt^2} + 2 \cdot \frac{dy}{dt} - 3y = \sin t$, 5

$y = \frac{dy}{dt} = 0$, when $t = 0$, using Laplace transform.