FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (New) (Common to all branches) (Main) Examination, June 2011

MATHEMATICS – I

Time : 3 Hours] **[Max. Marks : 75**

Note : Answer **all** questions from Part – **A**. Answer any **five** questions from Part – **B**.

PART – A (Marks : 25)

1. Using the Lagrange's mean value theorem, show that $|\sin b - \sin a| \le |b - a|$. **2**

2. Find the envelope of the family of curves $y = 3cx - c^3$, c is a parameter. **3**

3. If $f(x, y, z) = xy + yz + zx$, $x = t^2$, $y = te^t$, $z = te^{-t}$, find $\frac{df}{dt}$. **3**

4. Find the linear Taylor series polynomial approximation to the function $f(x, y) = 2x^3 + 3y^3 - 4x^2y$ about the point (1, 2). **2**

5. If $\overline{r} = xi + yj + zk$, show that $(\overline{u} \cdot \nabla)\overline{r} = \overline{u}$. **2**

6. Find the directional derivative of the function $f(x, y, z) = 2x^2 + y^2 + z^2$ at (1, 2, 3) in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$. **3**

7. Find the values of λ such that the rank of
$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & \lambda & 5 \\ 4 & 8 & \lambda \end{pmatrix}$$ is 2. **3**

8. Find the sum and the product of eigen values of the matrix $\begin{pmatrix} 10 & 0 & 8 \\ 4 & 9 & 6 \\ 2 & 7 & 5 \end{pmatrix}$. **2**

9. Find the values of x for which the series $\Sigma(4x)^n$ is convergent. **3**

10. Show that the series $\Sigma \frac{\sin nx}{n^2}$ converges absolutely. **2**

PART – B (Marks : 50)

11. (a) Find the radius of curvature of the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi$. **5**

    (b) Find the evolute of the curve $x = 2at$, $y = at^2$. **5**

12. (a) Trace the curve $r = a(1 + \cos\theta)$. 6

(b) Show that $f(x, y) = \begin{cases} \dfrac{x^2 + y^2}{x - y}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is not continuous at $(0, 0)$. 4

13. (a) Prove that $\text{curl}\,(f\,\overline{V}) = (\text{grad}\, f) \times \overline{V} + f\,\text{curl}\,\overline{V}$. 5

(b) Using Green's theorem, evaluate $\oint_C (x^2 + y^2)\,dx + (y + 2x)dy$, where C is the boundary of the region bounded by the curves $y^2 = x$ and $x^2 = y$. 5

14. (a) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, show that $A^n = A^{n-2} + A^2 - I$, $n \geq 3$ using Cayley-Hamilton theorem. 5

(b) Reduce $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ to the diagonal form. 5

15. Discuss the convergence of the series.

(a) $1 + \dfrac{2!}{2^2} + \dfrac{3!}{3^3} + \dfrac{4!}{4^4} + \ldots\ldots\ldots\ldots$ 4

(b) $\Sigma \dfrac{n^n x^n}{n!}$, $x > 0$. 6

16. (a) If $f(x, y) = \begin{cases} \dfrac{xy^3}{x + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$, compute $\dfrac{\partial^2 f}{\partial y\, \partial x}(0, 0)$. 4

(b) Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 6

17. (a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)}\,dx\,dy$. 5

(b) Test whether the vectors (1, 1, 0, 1), (1, 1, 1, 1), (4, 4, 1, 1), (1, 0, 0, 1) are linearly independent or not. 5
