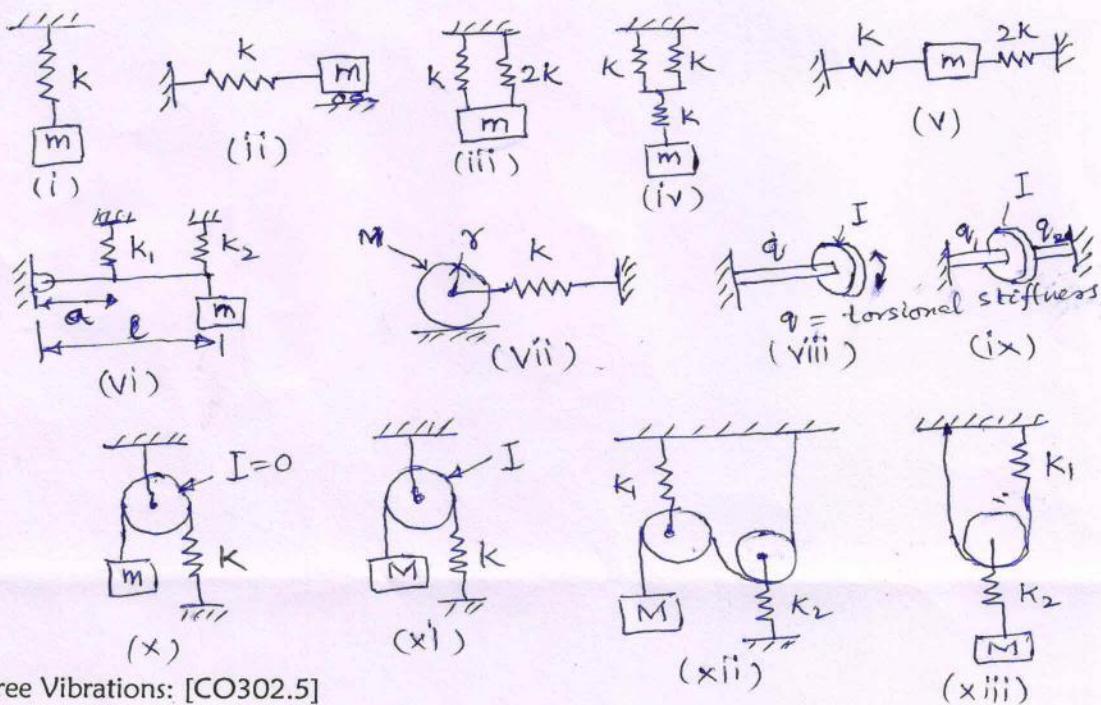


ASSIGNMENT: Problems - 1, 2, 3, 4, 5, 8, 9, 12, 14, 15, 16, 18

1. Find the frequency of free vibrations for the following cases (Quiz/short Answer)



Free Vibrations: [CO302.5]

2. For the system shown in Fig.1, find (i) Natural frequency (ii) Damped natural frequency (iii) Logarithmic decrement (iv) If the mass is initially displaced by 5 cms and the initial velocity is 30 cms/sec, find the displacement of mass at time, $t = 0.1$ secs.
(Data: $m=20$ kg; $k=30K$ N/m; $c=400$ Ns/m)
3. For the system shown in Fig.1, damping constant c is unknown. When undergoing free vibration, if the amplitude of vibration of mass reduces to $\frac{1}{4}$ in 3 oscillations, find the damping constant. (Other data as in above problem)

Forced Vibrations: [CO302.5]

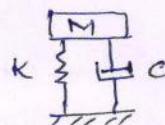


Fig.1

4. For the system shown in Fig.1, if a vertical harmonic force $3000 \sin \omega t$ acts on the mass, find the amplitude of vibration of the mass when (i) $\omega=30$ rad/sec and (ii) $\omega=80$ rad/sec.
5. For the above problem find the force transmitted to the base at (i) $\omega=30$ rad/s and (ii) $\omega=80$ rad/s.
6. For a system shown in Fig.2, the base is vibrating with a harmonic motion $y=0.05 \sin \omega t$. Find the amplitude of vibration of the mass when (i) $\omega=25$ rad/sec and (ii) $\omega=60$ rad/sec. (data: $m=50$ kg, $k=20K$ N/m; $c=400$ NS/m)

Fig.2

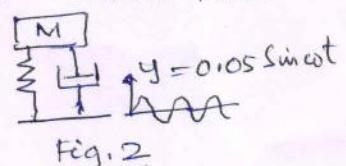


Fig.2

7. Static deflection of an automobile on its springs is 8 cms. Find the critical speed when travelling (Fig. 2) on a road which has undulation that can be approximated as a sine wave of 12 cms amplitude and wavelength of 3m. (damping ratio = 0.05). Also find amplitude of vibration at 60 kmph.
8. A vertical air compressor of 500 Kg mass is mounted on springs of $K = 196 \text{ k N/m}$. and dashpots of damping ratio 0.2. The reciprocating parts weigh 20 kg and stroke=0.2 m. Find amplitude of vertical motion and excitation force if compressor is operated at 200 rpm.

Whirling / Critical Speed ($E = 2 \times 10^{11} \text{ N/m}^2$)

9. A rotor 10 kg rotor is mounted midway on a 800 mm long 20 mm dia shaft. The CG of rotor is 0.1 mm away from its geometric centre. If system rotates at 50 rps, find the amplitude of vibration and dynamic load on bearings. (CO 502.5)
10. A vertical shaft 30 mm dia and 1 m long is mounted on long bearings and carries a pulley of 10 kg midway. CG of pulley is 0.5 mm away from shaft axis. Find (i) whirling speed (ii) bending stress in shaft at 2000 rpm.
11. Rotor of a turbocharger of 9 kg mass is keyed centrally to a shaft of 25 mm dia, 40cm length between bearings. Shaft material density = 8 gm/cm³; and shaft may be treated as simply supported. Find (i) critical speed (ii) Amplitude of rotation of rotor at 3200 rpm if eccentricity of rotor CG = 0.015 mm (iii) vibratory force transmitted to base.
12. A vertical steel shaft 15 mm dia is held in long bearings 1 m apart. It carries in its middle a disc of 15 kg. Eccentricity of disc is 0.3 mm. If the permissible tensile stress is 70 MN./m², find (i) critical speed of shaft (b) range of speeds unsafe to run. Neglect mass of shaft.

Multi-rotor Systems : (Rigidity Mod., $N = 80 \text{ GPa}$)

13. Find the natural freq., mode shape and node of the 2-rotor system.

Given $I_A = 20 \text{ kg.m}^2$; $I_B = 10 \text{ kg.m}^2$; $d = 50 \text{ mm dia}$; $l = 900 \text{ mm}$

14. Find the natural freq., mode shape and node of the 2-rotor system with stepped shaft. .

Given $I_A = 20 \text{ kg.m}^2$; $I_B = 10 \text{ kg.m}^2$;

$l_1 = 600 \text{ mm}$; $l_2 = 300 \text{ mm}$; $d_1 = 30 \text{ mm dia}$; $d_2 = 25 \text{ mm dia}$.

15. For the 3 rotor system, find the natural freq., mode shapes and the nodes, given

$l_1 = 600 \text{ mm}$; $l_2 = 300 \text{ mm}$; $d = 50 \text{ mm dia}$;

$I_A = 20 \text{ kg.m}^2$; $I_B = 10 \text{ kg.m}^2$; $I_C = 8 \text{ kg.m}^2$

16. For the 2-mass system shown, find the natural frequencies and mode shapes.

$m_1 = 30 \text{ kg}$; $m_2 = 15 \text{ kg}$; $k_1 = 20 \text{ KN/m}$; $k_2 = 10 \text{ KN/m}$

Dunkerley's Method:

17. A cantilever beam with uniformly distributed mass has a natural frequency of lateral vibration of 20 cps. If a 25 kg mass is attached at the end of the cantilever, the end deflects by 15 mm. Find the natural frequency of the beam along with the mass, using Dunkerley's equation.

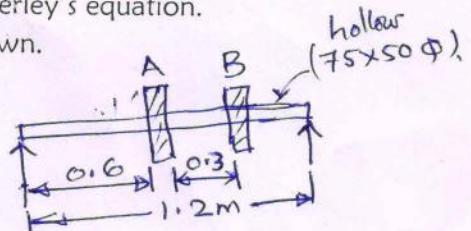
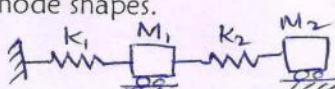
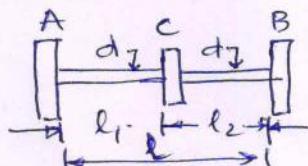
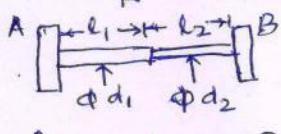
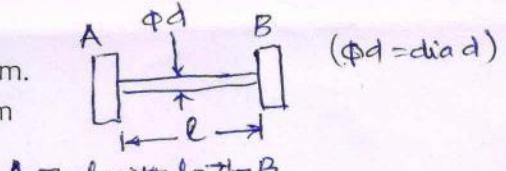
18. A simply supported shaft 1.5 m long at ends carries 2 wheels as shown.

Shaft is hollow with 75x50 mm dia. Density of shaft material

is 7800 kg/m^3 . Find the frequency of transverse vibration

(i) Neglecting shaft mass and (ii) Including shaft mass.

(data: $m_A = 60 \text{ kg}$; $m_B = 100 \text{ kg}$; $E_{\text{shaft}} = 200 \text{ GPa}$)



Qn. 1. (i) $\omega_n = \sqrt{k/m}$; (ii) $\sqrt{\frac{k}{m}}$ (iii) $\sqrt{\frac{k+2K}{M}}$ (iv) $\sqrt{\frac{3k/2}{M}}$

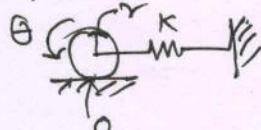
(v) $\sqrt{\frac{k+2K}{M}}$ (viii) $\sqrt{\frac{q}{I}}$ (ix) $\sqrt{\frac{q_1+q_2}{I}}$ (x) $\sqrt{\frac{K}{M}}$

(VII) Deflection of M = deflection of (due to k_1 + due to k_2)

$$\therefore \delta = \frac{(mg \times \frac{l}{a})}{k_1} \times \frac{l}{a} + \frac{mg}{k_2} = mg \left(\frac{l^2/a^2}{k_1} + \frac{1}{k_2} \right)$$

$$\omega = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{a^2 k_1 k_2}{m(l^2 k_2 + a^2 k_1)}}$$

(vii) The roller oscillates on ground, and rotates about the point of contact O.



$$x = r\theta \quad \& \quad \ddot{x} = r\ddot{\theta}$$

$$\text{Equating torques about } O, \quad I_O \ddot{\theta} = (-Kx) \times r$$

$$\text{or } \left(\frac{3}{2}Mr^2\right) \frac{\ddot{\theta}}{r} \neq Kx \cdot r = 0$$

$$I_O = I + mr^2 \\ = \frac{mr^2}{2} + mr^2 \\ = \frac{3}{2}mr^2$$

$$\Rightarrow \omega = \sqrt{\frac{2K}{3m}}$$

(xi)



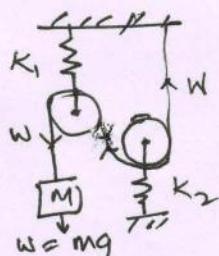
$$F_1 = -m\ddot{x}; \quad F_2 = Kx; \quad (F_1 - F_2)r = I\ddot{\alpha} = I\ddot{\theta} = I\frac{\ddot{x}}{r}$$

$$\rightarrow (-m\ddot{x} - Kx)r = I\ddot{x}/r$$

$$\rightarrow \ddot{x} + \frac{Kr^2}{Mr^2 + I} x = 0 \rightarrow \omega_n = \sqrt{\frac{Kr^2}{Mr^2 + I}}$$

(Note: If $I = \frac{Mr^2}{2}$ is used, $\omega_n = \sqrt{\frac{K}{(m+M/2)}}$)

(xii)

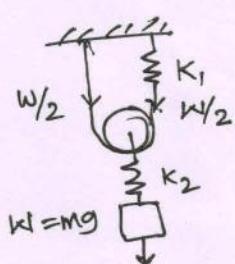


The mass deflects by 2 times the extension of spring connected to pulley. Load on each spring = 2w.

$$\delta_{\text{mass}} = 2\delta_{K_1} + 2\delta_{K_2} = 2\left(\frac{w}{K_1} + \frac{w}{K_2}\right) \quad (w = mg)$$

$$\Rightarrow \omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{K_1 K_2}{2m(K_1 + K_2)}}$$

(xiii)



$$\delta_{\text{mass}} = \frac{\delta_{K_1}}{2} + \delta_{K_2} = \left(\frac{w/2}{K_1}\right)/2 + \frac{w}{K_2}$$

$$\Rightarrow \omega = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{4K_1 K_2}{m(K_2 + 4K_1)}}$$

Qn.2: (i) $\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{30000}{20}} = 38.7 \text{ rad/s}$; $\xi = \frac{C}{C_c} = \frac{400}{2\sqrt{Km}} = 0.258$.

(iii) $\omega_d = \omega_n \sqrt{1-\xi^2} = 37.4 \text{ rad/s}$; $\delta = \ln \frac{x_1}{x_2} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 1.68$.

(iv) For free damped vibrations,

$$x(t) = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi) \quad (1)$$

$$\rightarrow x(0) = X \sin \phi = 0.05 \text{ (given)} \quad (A)$$

$$\dot{x}(t) = X e^{-\xi \omega_n t} (\omega_d \cos(\omega_d t + \phi) + (-\xi \omega_n) \sin(\omega_d t + \phi))$$

$$\dot{x}(0) = 0.3 = X (\omega_d \cos \phi - \xi \omega_n \sin \phi) \quad (B)$$

$$\text{From A and B, } \phi = 23.14^\circ \text{ & } X = 0.127$$

$$\text{Eqn (1)} \rightarrow x(t) = 0.127 e^{-9.985t} \sin(37.4t + 23.14^\circ)$$

$$\text{from which } x(0.1) = 0.0212 \text{ m} = \boxed{21.2 \text{ mm}}$$

Qn.3

$$\frac{x_4}{x_1} = \frac{1}{4}; \frac{x_4}{x_3} \cdot \frac{x_3}{x_2} \cdot \frac{x_2}{x_1} = \frac{\delta}{2} \cdot \frac{\delta}{e} \cdot \frac{\delta}{e} = e^{3\delta} = \frac{1}{4},$$

$$3\delta = \ln 4 \Rightarrow \delta = 0.462;$$

$$\delta = 2\pi\xi \rightarrow \xi = 0.0735 = \frac{C}{C_c} = \frac{C}{2m\omega_n}$$

$$0.0735 = \frac{C}{2 \times 20 \times 38.7} \rightarrow \boxed{C = 113 \text{ NS/m}}$$

Qn.4

$$\frac{x}{x_0} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \quad (1) \quad \text{in which,}$$

$$\xi = 0.258; \omega_n = 38.7; x_0 = \frac{F_0}{K} = \frac{3000}{30000} = 0.1$$

$$(i) \& r = 30/38.7 \quad (\text{ie } \omega/\omega_n) = 0.775$$

$$\text{Eqn (1) gives } x = 0.177 \text{ m} = \boxed{17.7 \text{ cm}}.$$

$$(ii) r = 80/38.7 = 2.067; \text{ Eqn. (1) gives } x = 0.029 \text{ m} = \boxed{2.9 \text{ cm}}$$

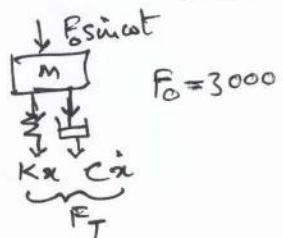
Qn.5:

$$F_T = \text{Force transmitted to base} = F_0 \cdot \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)+(2\xi r)^2}}$$

$$(i) \omega = 30; r = 0.775 \rightarrow F_T = 3000 \times 1.906$$

$$(ii) \omega = 80; r = 2.067 \rightarrow F_T = \underline{5718 \text{ N}}$$

$$\rightarrow F_T = 3000 \times 0.425 \\ = \underline{1275 \text{ N}}$$



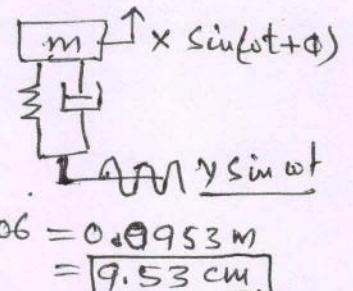
Qn. 6: Amplitude of vibration of the mass (x) to base motion amplitude (y) is related by the

Same formula as force transmitted to base,

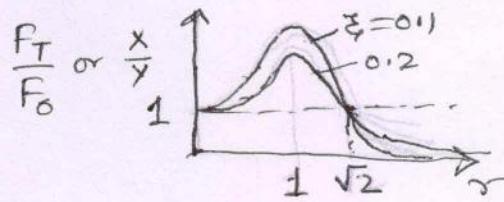
$$\frac{x}{y} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$(i) \omega = 30; r = 0.775; x = y \times 1.906 = 0.05 \times 1.906 = 0.0953 \text{ m}$$

$$(ii) \omega = 80; r = 2.067; x = 0.05 \times 0.425 = 0.02125 \text{ m} = 2.125 \text{ cm}$$



Note: In the case of Base motion and force transmitted to base the frequency response curves are similar, and intersect at $r = \sqrt{2}$ and lower damping is better when forcing frequency $> \sqrt{2} \omega_n$ (unlike the case in Qn. 4).



Qn. 7: Automobile case is similar to base vibration case

$$\delta = 8 \text{ cms} = 0.08 \text{ m}; \omega_n = \sqrt{\frac{g}{\Delta}} = 11.07 \text{ rad/s}$$

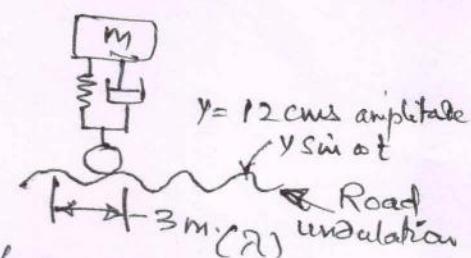
If speed is such that induction frequency matches with natural frequency of automobile ($\sqrt{k/m}$ or g/Δ), then violent vibration occurs due to resonance. That

- Critical speed = $f \times \lambda = 1.762 \times 3 = 5.285 \text{ m/s} = 19.03 \text{ kmph.}$

$$\text{and } r = \frac{5.56}{1.762} = 3.156, f = \frac{V}{\lambda} = 5.56 \text{ cps.}$$

Automobile movement (vertical) amplitude = $y = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} (ξ=0.05)$

$$= 0.12 \times 0.117 \Rightarrow 1.44 \text{ cms.}$$



Qn. 8:

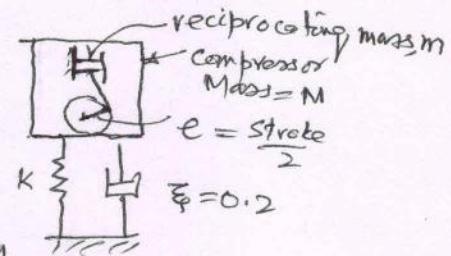
Amplitude of compressor is given by

$$\frac{x}{(me/M)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

where $M = 500 \text{ kg}$, $m = 20 \text{ kg}$; $e = \frac{0.2}{2} = 0.1 \text{ m}$

$$\gamma = \frac{\text{Exciting freq.}}{\text{Nat. freq.}} = \frac{(200 \times 2\pi/60)}{\sqrt{k/m}} = 1.057$$

$$\Rightarrow \frac{x}{(me/M)} = 2.546 \quad \text{or } x = 2.546 \times \frac{20 \times 0.1}{500} = 0.0102 \text{ m}$$



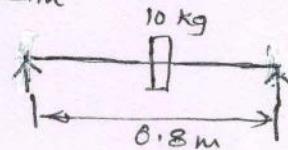
$$k = 196000 \text{ N/m}$$

$$= 10.2 \text{ mm}$$

Qn. 9

Taking the shaft as a simply supported beam

$$\delta = \frac{Wl^3}{48EI} = \frac{10 \times 9.81 \times 0.8^3}{48 \times 2 \times 10^{11} \times \pi \times 0.02^4} = 6.66 \times 10^{-4} \text{ m.}$$



Nat. frequency of lateral vibration = $\sqrt{g/\delta} = 121.4 \text{ r/s.}$

Shaft speed = 50 rpm = $100\pi \text{ r/s.}$; $r = \frac{100\pi}{121.4} = 2.588.$

$$\text{Using the relation } x = \epsilon \frac{r^2}{1-r^2} = (0.1 \times 10^{-3}) \times (-1.176) = -1.176 \times 10^{-4} \text{ m.}$$

Total Dynamic load on bearings = $k \cdot x = \frac{W}{8} \times x$

$$= \frac{10 \times 9.81}{6.66 \times 10^{-4}} \times 1.176 \times 10^{-4} = 17.32 \text{ N}$$

Qn. 10: Treating as fixed beam, due to long bearings,

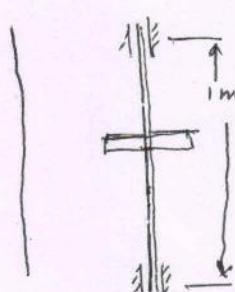
$$\delta = \frac{Wl^3}{192EI} = 6.425 \times 10^{-5} \text{ m.}$$

As in above problem, $\omega_n = \sqrt{g/l} = 390.75 \text{ r/s}$
 $= 3731 \text{ rpm.}$

$$\gamma = \frac{\omega}{\omega_n} = \frac{2000}{3731} = 0.536$$

$$\text{Lateral deflection } x = \epsilon \times \frac{r^2}{1-r^2} = 0.2016 \times 10^{-3} \text{ m.}$$

$$\text{Bending force} = kx = \frac{W}{8}x = \frac{10 \times 9.81}{6.425 \times 10^{-5}} \times 0.2016 \times 10^{-3} = 307 \text{ N}$$



Bending Moment, $M = Fl/8 = \frac{307}{8}$

From flexure formula, $f = \frac{M}{I} y = \frac{307}{8} \times \frac{1 \times 0.015}{\frac{\pi}{64} \times 0.034}$

Bending Stress on shaft = $[14.48 \text{ MPa}]$ at 2000 rpm.

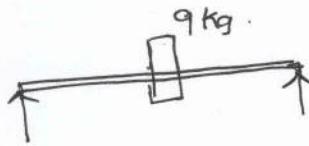
Qn. 11: As shaft matl. density is given,

ω of shaft is also considered.

ω_n of shaft and rotor is found from Danterley's eqn. as

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_{\text{shaft}}^2}$$

where ω_1 is nat. freq. due to 9 kg mass.



$$\text{we have } \omega_1 = \sqrt{\frac{g}{\delta_w}} \quad \& \quad \omega_{\text{shaft}} = \sqrt{\frac{g}{(\delta_{\text{shaft}}/1.27)}}$$

$$\delta_w = \frac{wl^3}{48EI} = \frac{9 \times 981}{48 \times 2 \times 10^{11} \times \frac{\pi d^4}{64}} = 3.07 \times 10^{-5} \text{ m.}$$

$$\delta_{\text{shaft}} = \frac{5}{384} \frac{wl^4}{EI} \quad \text{where } w \text{ est. per unit length of shaft}$$

$$(\text{where } w = \frac{\pi}{4} d^2 \times 1 \times 8000 \times g)$$

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_1^2 + (\delta_{\text{shaft}}/1.27)} = 0.335 \times 10^{-5} \text{ m}$$

$$\rightarrow \omega_n = 542 \text{ r/s} = [5176 \text{ rpm.}]$$

(i) Being the natural frequency of lateral deflection, it is the critical speed.

$$(ii) r = \frac{\omega}{\omega_n} = \frac{3200}{5176} = 0.696.$$

$$x = \frac{r^2}{1-r^2} = 0.015 \times 5.242 = [0.0786 \text{ mm.}]$$

is the amplitude of rotation of shaft

(iii) Force transmitted to bearings

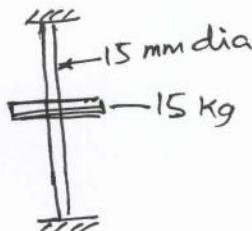
$$F = m\omega^2(x+r) = 9 \times \left(\frac{2\pi \times 3200}{60} \right)^2 \times (0.0786 + 0.015) \times 10^3 \\ = [94.6 \text{ N}]$$

Qn. 12: Treating as fixed beam,

$$\delta = \frac{wl^3}{192EI} = 1.542 \times 10^{-3} \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = 80 \text{ rad/sec.}$$

$$l = 1 \text{ m} \\ E = 2 \times 10^{11}$$



Next - use flexure formula $f = \frac{M}{I} y = \left(\frac{Fl}{8}\right) \frac{1}{I} y$

to find Force F reqd. to reach permissible stress (f)

$$\text{or } F = \frac{8If}{Ly} = \frac{8 \times \pi}{64} d^4 \times \frac{(70 \times 10^6)}{1 \times 0.015/2} = 185 \text{ N.}$$

- Corresponding deflection is $(x) \frac{185}{15 \times 9.81} \times 1.542 \times 10^{-3} = 1.939 \times 10^{-3} \text{ m.}$

$$\frac{x}{e} = \frac{1.939 \times 10^{-3}}{0.015} = \frac{r^2}{1-r^2} = 6.463$$

Choosing +ve & -ve values of x, $\pm 6.463 = \frac{r^2}{1-r^2} = \frac{1}{r^2-1}$
gives the values of $\gamma = 0.93 \omega_n = 0.93 \times 80 \text{ rad/s}$

$$\Rightarrow 710 \text{ rpm}$$

$$\gamma = 1.087 \omega_n \Rightarrow 830 \text{ rpm.}$$

In the range of speeds 710-830 rpm, the shaft bends so much that tensile stress on the shaft exceeds. permissible value of $70 \times 10^6 \text{ N/m}^2$ should be outside the range of 710-830 rpm.

Qn. 13

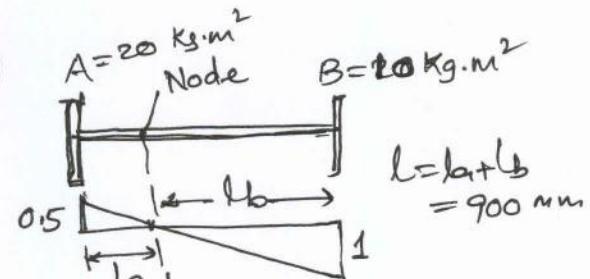
Equating nat. freq. of A and B about the node,

$$\omega_{nA} = \omega_{nB} \Rightarrow \sqrt{\frac{N_J}{I_{la}}} = \sqrt{\frac{N_J}{I_b l_b}}$$

$$\Rightarrow I_{la} = I_b l_b$$

$$\frac{l_a}{l_b} = \frac{I_b}{I_a} = \frac{10}{20} = \frac{1}{2} \quad \text{or} \quad \frac{l}{l_b} = \frac{3}{2} \rightarrow l_a = 300, l_b = 600 \text{ mm.}$$

$$\frac{A_a}{A_b} = \frac{l_a}{l_b} = \frac{300}{600} = \frac{1}{2} : \quad \omega_n = \sqrt{\frac{N_J}{I_{la}}} = 14.47 \text{ rad/s}$$



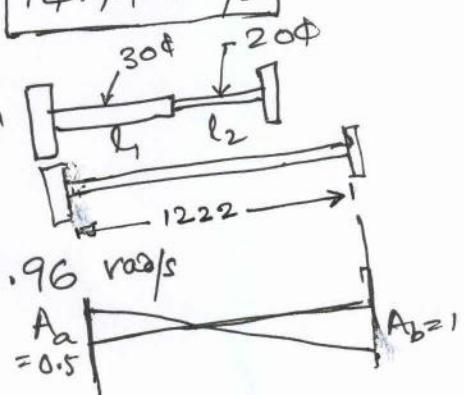
Qn. 14: Stepped shaft.

$$l_{30^\circ} (\text{equivalent}) = l_1 + l_2 \times \left(\frac{30}{20}\right)^4 = 600 + 622 = 1222 \text{ mm}$$

$$I_{la} = I_b l_b \Rightarrow l_a = 407 \text{ mm.}$$

$$\omega_n = \sqrt{\frac{N_J}{I_{la}}} = \sqrt{\frac{80 \times 10^9 \times \frac{\pi}{32} \times 0.03^4}{20 \times 0.407}} = 27.96 \text{ rad/s}$$

$$\frac{A_a}{A_b} = \frac{l_a}{l_b} = \frac{407}{997} = \frac{407}{815} = 0.4993 \approx 0.5$$



Qn. 15:

$$\omega_a = \omega_b = \omega_c$$

$$\frac{N_J}{I_{ala}} = \frac{N_J}{I_{blb}} = \frac{N_J}{I_c} \sqrt{\frac{1}{l_1-l_a} + \frac{1}{l_2-l_b}}$$

$$l_a = l_b/2 \quad (\text{from } I_{ala} = I_{blb})$$

$$\& \frac{1}{I_{ala}} = \frac{1}{I_c} \left(\frac{1}{l_1-l_a} + \frac{1}{l_2-2l_a} \right)$$

$$\Rightarrow 9.5 l_a^2 - 3.75 l_a + 0.18 = 0$$

$$\therefore l_a = 0.339 \quad \& \quad 0.056$$

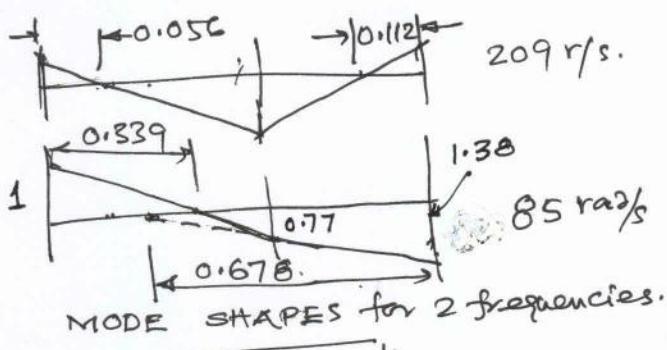
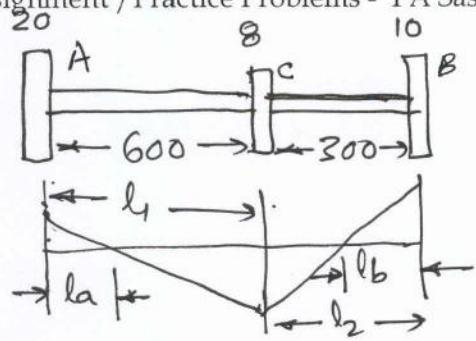
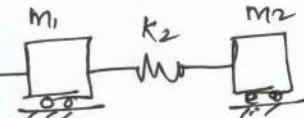
$$\therefore l_b = 0.678 \quad \& \quad 0.112$$

$$\frac{A_c}{A_A} = \frac{0.6 - 0.339}{0.339} = 0.77$$

$$\frac{A_B}{A_c} = \frac{0.678}{0.678 - 0.3} = 1.79$$

$$AB = 1.38 \quad \& \quad \omega_{n1} = \sqrt{\frac{N_J}{I_{ala}}} = \sqrt{\frac{80 \times 10^9 \times \frac{\pi}{32} \times 0.05^4}{20 \times 0.339}} = 85 \text{ rad/s}$$

$$\therefore \omega_{n2} = 209 \text{ rad/s}$$


 Qn. 16: We can use the general frequency equation, and make $K_3=0$;


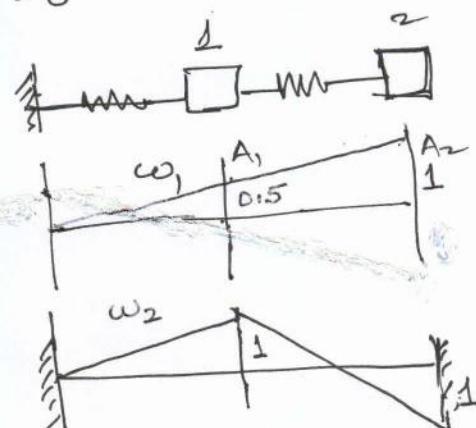
$$\rightarrow \omega^4 - \left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \left(\frac{k_1 k_2}{m_1 m_2} \right) = 0$$

$$\Rightarrow \omega^4 - 1667 \omega^2 + 44414 = 0$$

$$\omega_1 = \sqrt{334} \quad \& \quad \omega_2 = \sqrt{1334}$$

$$\& \left(\frac{A_1}{A_2} \right)_1 = \frac{k_2}{k_1 + k_2 - m_1 \omega^2} = \frac{10000}{30000 - 80 \times 334} = 0.5$$

$$\left(\frac{A_1}{A_2} \right)_2 = -1$$



Q17

BE III Mech. (A&B) V Sem 2018 D.O.M - Hints / Solutions to Assignment / Practice Problems - PA Sastry

$$\omega_n (\text{due to mass}) = \sqrt{\frac{9.81}{\Delta}} = \sqrt{\frac{9.81}{15 \times 10^{-3}}} = 25.57 \text{ r/s} \\ = 4.07 \text{ cps.}$$

$$\frac{1}{f_n^2} = \frac{1}{f_{\text{beam}}^2} + \frac{1}{f_{\text{mass}}^2} \Rightarrow \frac{1}{f_n^2} = \frac{1}{20^2} + \frac{1}{4.07^2} \\ \Rightarrow f_n = \boxed{4.017 \text{ cps}}$$

Q18

$$\delta_A = \frac{W a^2 b^2}{3EI l} = \frac{60 \times 9.81 \times 0.6^2 \times 0.6^2}{3 \times 2 \times 10^{11} \times 124.64 \times 10^8 \times 1.2} \\ = 8.5 \times 10^{-5} \text{ m}$$

$$\delta_B = 7.97 \times 10^{-5}$$

Nat. frequency due to shaft mass

$$= \omega_s = \pi^2 \sqrt{\frac{EI}{m l^4}} = 779 \text{ r/s.}$$

Using Dunkerley's equation,

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_s^2} \\ = \frac{1}{g/\delta_A} + \frac{1}{g/\delta_B} + \frac{1}{779^2} =$$

$$\Rightarrow \omega_n = \boxed{223 \text{ r/s}} \leftarrow \text{including shaft mass}$$

ω_n without shaft mass,

$$\frac{1}{\omega_n^2} = \frac{1}{g/\delta_1} + \frac{1}{g/\delta_2} = \frac{\delta_1 + \delta_2}{g} \Rightarrow \omega_n = \boxed{294 \text{ r/s.}}$$

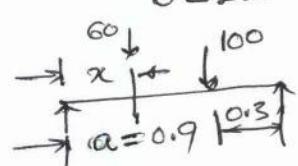
Raleigh's Method : (neglecting shaft mass)

$$\text{Equating PE} = \text{KE} \rightarrow \frac{1}{2} (m_1 g y_1 + m_2 g y_2 + \dots) = \frac{1}{2} (m_1 y_1^2 + m_2 y_2^2 + \dots) \omega^2$$

$$(A) \dots \omega_n^2 = \frac{g(m_1 y_1 + m_2 y_2 + \dots)}{m_1 y_1^2 + m_2 y_2^2} \quad \text{in which each deflection is taken}$$

as due to load at that point + due to other loads.

$$y_1 = \omega_1 \delta_{11} + \omega_2 \delta_{12} \quad \text{where } \delta_{11} = \frac{a^2 b^2}{3EI l} \quad \delta_{12} = \frac{b x (l^2 - a^2 - b^2)}{6EI l} \\ = \frac{60 \times 0.6^2 \times 0.6^2}{3EI l} + \frac{100 \times 0.3 \times 0.6^2 (1.2^2 - 0.9^2 - 0.3^2)}{6EI l} \\ = 13.81 \times 10^{-5} \text{ m}; \quad \text{and } y_2 = 11.16 \times 10^{-5}$$



Substituting in Eqn. (A), $\omega_n = 282 \text{ r/s}$ and
is more accurate than Dunkerley's method which gave $\omega_n = 294 \text{ r/s}$, (See above)

$\delta_{12} = \delta_{21}$
(influence coefficient)