Linearization

Most systems have non-linear behavior. When the behavior is represented by a smooth curve, and not sudden changes, linear approximations can be effectively employed in control theory.

We use Taylor's approximation as follows:

Let y = $4x^2$ + 3x + 12

Exact values of y at x=10 and x=11 are

y_{10} = $4x^2$ + 3x + 12 = 4.100 + 3.10 + 12 = 442

y_{11} = 4.11^2 + 3.11 + 12 = 484 + 33 + 12 = 529

Suppose we wish to use the system model where x=10

We will find y_{11} by linear approximation

$$Y = f(x_0) + \left(\frac{df}{dx}\right)(x - x_0) + \frac{d^2f}{dx^2}\left(\frac{(x-x_0)^2}{2!}\right.$$

$$\cong f(x_0) + \left(\frac{df}{dx}\right)(x - x_0)$$

Apply this for the given equation to find y_{11} when the neighbouring value y_{10} is known

$$y_{11} = y_{10} + \frac{dy}{dx}.(x - x_0)$$

$$\frac{dy}{dx} = 8x + 3 = 8.10+3 = 83$$

$$y_{11} = y_{10} + 83\ .(11-10)$$

$$= 442 + 83 = 525$$

Which is close to the actual value of 529, and the linear approximation is acceptable.

Linearization for a function of two variables

Process is similar, except that Taylor's formula for the 2-variable case is used.

$$Y = f(x,y) = f(x_0, y_0) + \frac{df}{dx}(x - x_0) + \frac{df}{dy}(y - y_0)$$

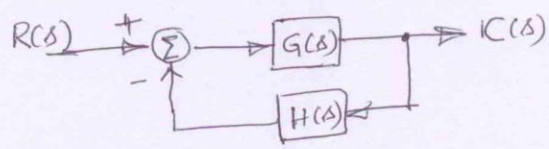
or $y = y_0 + K_1\,\Delta x + K_2\,\Delta y$

leaving out 2nd order differential terms. Thus we can linearize a non-linear function of two variables about the point (x_0, y_0).

Advantages of feedback

1) Output tracking: In open-loop, an input R is given from the knowledge of G(s) and an output is obtained. If C is different, for any reason, due to error in G(s) model or its variation due to changes in surroundings, C cannot be corrected.
   In closed-loop control error is feedback to plant G(s) if C is different from that expected, Error drives the plant G(s) till C is the expected value for given R. Hence output is always improved, even if the parameters are varied.
2) System Response: System dynamics can be improved in a CL system.
   For OL, C(s)/R(s) = k/s+a
   Time constant, T = 1/a, & d.c. gain (s=0) is k/a

   For CL case, C(s)/R(s) $=k_1/k_1+s+a$
   For R(s) =1, C(s) = k_1/k_1+s+a; c(t) = $k_1\, e^{-(a+k_1)t}$
   T = $1/a+k_1$ & dc gain = $k_1/(k_1+a)$
   Thus T, or speed of response can be controlled by varying K.
   (Reduction of DC gain can be compensated by additional gain outside the loop)
3) Sensitively to variation in parameters
   Let the overall transfer function be T(s)



   for OL, T(s) = G(s); CL → T(s) = G/1+GH
   Sensitively can be described as
   $S_G^T = \left(\frac{\partial T}{T}\right) / \left(\frac{\partial G}{G}\right) = \frac{\partial T}{\partial G} \cdot \frac{G}{T}$
   S_G^T = 1 for O.L system
   For CL system, $S_G^T = \left(\frac{\partial T}{\partial G}\right) \cdot \frac{G}{T}$ = ((1+GH) – GH)/$(1+GH)^2$. (G/(G/1+GH)) = 1/(1+GH)
   Thus while it is 1 for OL system, it is 1/(1+GH) for CL system. CL system is less sensitive to variation in parameters.
   CL system is also sensitive to variation in H. But the feedback elements are less costly and controlled more easily.