

Static and Dynamic Force Analysis of Mechanisms

Mechanisms are designed to carry out certain desired work, by producing the specified motion of certain output member. It is usually required to find the force or torque to be applied on an input member, when one or more forces act on certain output member(s). The external force may be constant or varying through the whole cycle of motion. Calculation of input force or torque over the complete cycle will be needed to determine the power requirement. When the masses and moments of inertia of the members are negligible, static force analysis may be carried out. Otherwise, particularly at high speeds, significant forces or torques will be required to produce linear or angular accelerations of the various members. The same will have to be considered in the analysis. It is also required to find the forces at the joints for proper design. These also vary depending upon the position/configuration in the cycle.

Static analysis is carried out by the usual methods of collinearity of forces, equilibrium of forces / moments. Input is determined as that force or moment to bring the system to equilibrium. In the case of dynamic systems, linear acceleration of each link (CG) and the angular accelerations of the members are evaluated. The corresponding forces and moments are calculated (product of acceleration and inertia).

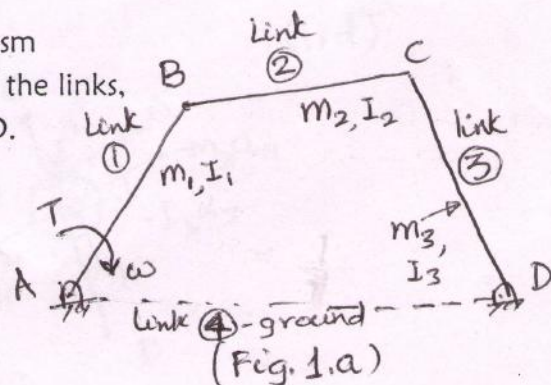
D'Alembert's principle is a method of applying fictitious forces / torques called inertia force / torque, equal and opposite to the force or torque required to produce acceleration in each member, so as to produce a static condition which is called dynamic equilibrium. Then the system can be treated as static, which permits application of techniques of static force analysis.

Dynamic force analysis is the evaluation of input forces or torques and joint forces considering motion of members. Evaluation of the inertia force /torque are explained first. Methods of static force analysis are explained.

Dynamic Force Analysis:

Consider the four-bar chain ABCD (fig.1a). Let the joint A be acted upon by a Torque T so as to move the link AB at an angular velocity of ω . Let the masses of the links AB, BC and CD be m_1 , m_2 and m_3 , and moments of inertia be I_1 , I_2 , and I_3 .

- Draw the velocity (fig 1.b) and acceleration diagram (fig 1.c) of the mechanism
- Determine linear accelerations of the CGs of the links, and angular accelerations of links BC and CD.



iii. Consider link BC. Let the CG be at point G. (fig. 1.d)

Force on the link due to acceleration a_2 is, $f_2 = m_2 \times a_G$

Hence Inertia force = $-f_2$

Angular acceleration = $\alpha_2 = a_2^t / BC$;

Torque $t_2 = I_2 \times \alpha_2$. (ccw)

Inertia torque = $-t_2$ (cw)

iv. Combine the inertia force and torque into a single force P, parallel to it, but acting at distance $h = I_2 \alpha_2 / m_2 a_2$ from the point G. (Fig.1.d)(Verify)

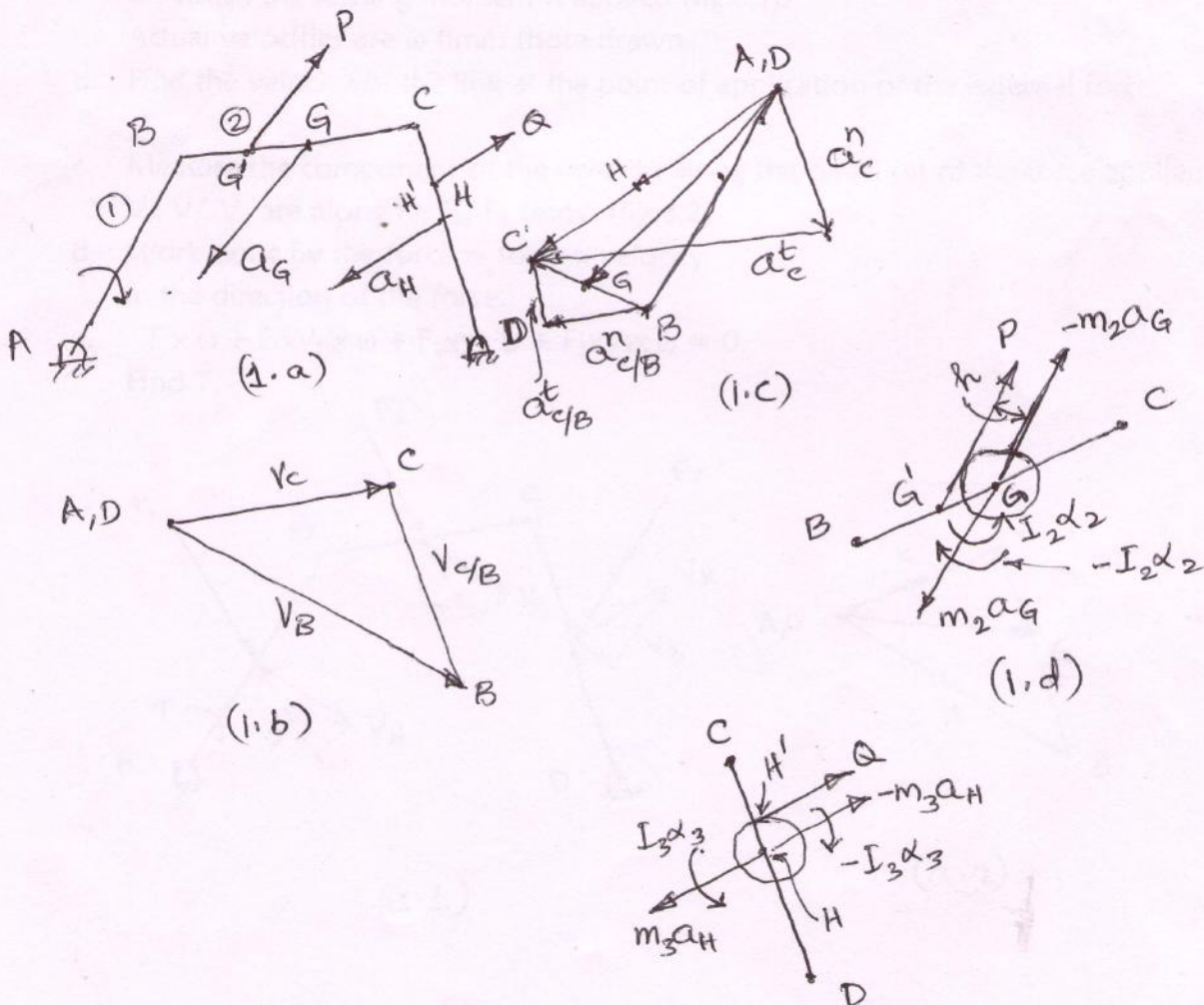
v. This force equivalently replaces the inertia force and torque.

vi. Repeat the procedure for link CD. (fig.1.e)

vii. For link AB, as there is no angular acceleration, inertia force is taken to act opposite to $m_1 a_1$. (If it has finite angular acceleration, given as input, it can be handled as for other links)

viii. Thus, the mechanism will be in equilibrium under the action of the forces acting on links 2 and 3 and the input torque. It is then a static system.

The torque on the crank is calculated by any of the methods of static force analysis, some of which are explained below. later.



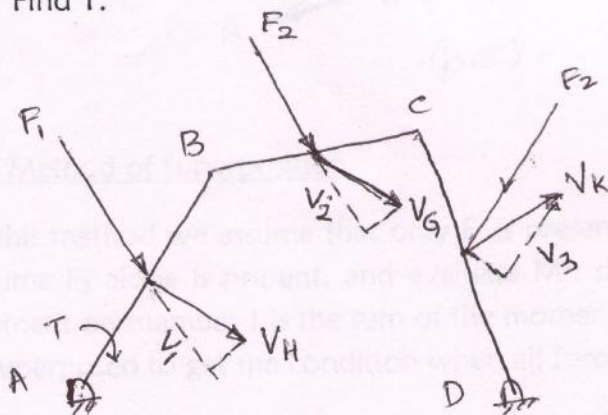
Static Force Analysis:

This can be done by obtaining the free body diagrams (f.b.d.) for each link, application of equilibrium of forces or moments and collinearity of forces, as appropriate. Either graphical-analytical methods or vectorial approach can be adapted. We review (a) Principle of Virtual Work (b) method of force resolution and (c) Method of superposition. (We may also employ equivalent vectorial methods – see JE Shigley).

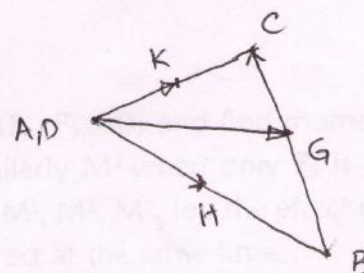
Consider a 4-bar chain. Forces F_1 , F_2 , F_3 act on links 1, 2 and 3 at the points shown. It is desired to find the torque T on link 1, (and joint forces) to keep the mechanism in equilibrium.

A.Principle of Virtual Work: In this method, total work done by forces and moments acting on the system causing infinitesimal motions, is taken as zero. It is to be noted that the reactions at the joints get nullified and are workless. As such the joint forces cannot be evaluated in this method. Following procedure is adapted:

- Draw a velocity diagram of the linkage assuming unit angular velocity of the link AB on which the turning moment is applied (fig.a.1).
Actual velocities are ω times those drawn.
- Find the velocity of the link at the point of application of the external force.
- Measure the component of the velocity along the direction of the force applied.
 V_1, V_2, V_3 are along F_1, F_2, F_3 respy. (fig.a.2)
- Work done by the force = force x velocity in the direction of the force.
- $T \times \omega + F_1 \times V_1 \times \omega + F_2 \times V_2 \times \omega + F_3 \times V_3 \times \omega = 0$.
- Find T .



(a.1)



(a.2)

B : By Resoluion of Forces:

Start with link 3.

-From the fbd of link 3, let the force f_{23} be resolved into two components, one along Link 3 and other perpendicular. (fig.b.1)

-Take moments about D, which gives f_{23}^t

Link 2

- F_2 and F_{32}^t being known, taking moments about B, find f_{32}^n . (fig.b.2)

-From polygon of forces, find f_{12} (fig.b.3)

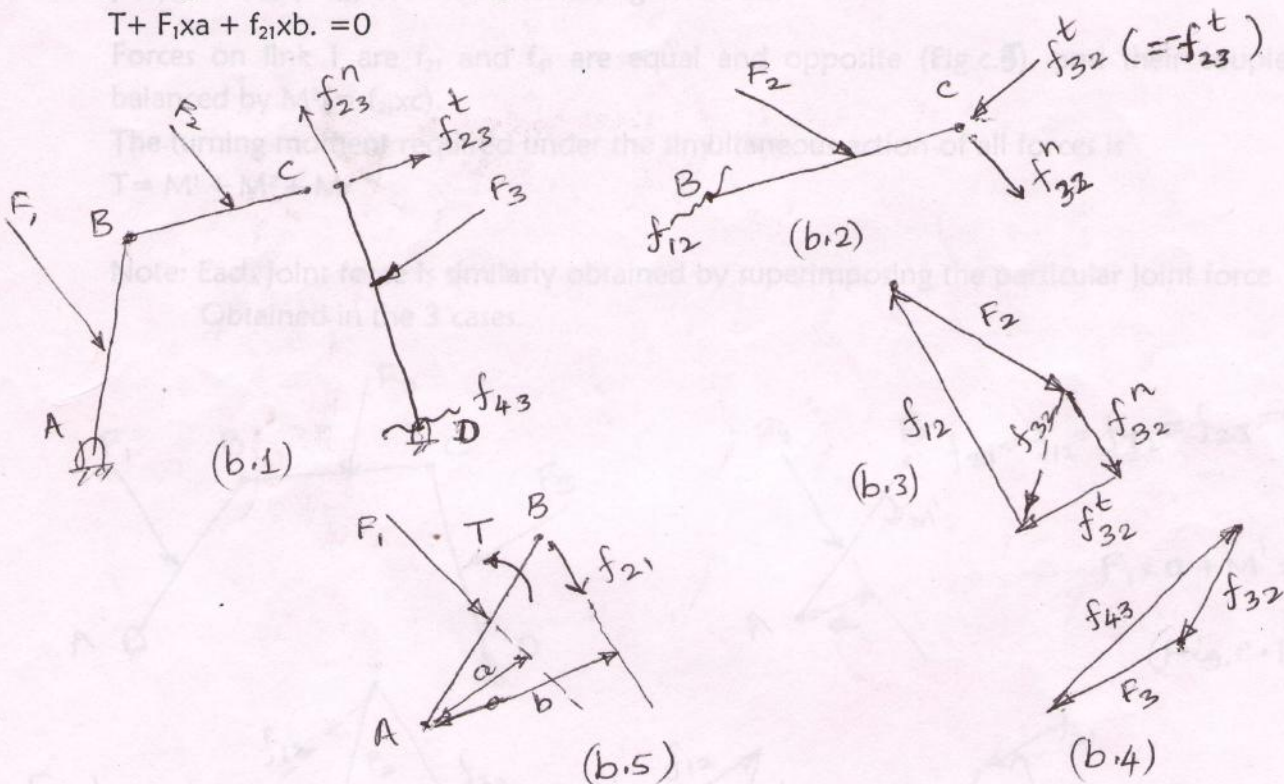
- F_3 and f_{23} components being known, force polygon gives f_{43} . (fig.b.4).

Link 1

From the polygon of forces on link 1, find f_{41} . (not shown)

Taking moments about A, (fig.b.5), find T from the eqn.

$$T + F_1 x_a + f_{21} x_b = 0$$



(C) Method of Superposition

In this method we assume that only F_1 is present ($F_2, F_3 = 0$) and find moment M^1 . Then assume F_2 alone is present, and evaluate M^2 , similarly M^3 when only F_3 is present. The moment on member 1 is the sum of the moments M^1, M^2, M^3 . i.e., the effect of each force is superposed to get the condition when all forces act at the same time.

(a) Effect of F_1 alone (fig.c.1): Start with the fbd for link 1 - links 2 and 3 are 2-force members, and joint forces are along the members. However, at joint C, force f_{23} and f_{32} act along the respective members 2 and 3, but have to be equal and opposite. It is possible only $f_{23} = f_{32} = 0$. Hence, $f_{21}(=f_{12})$ and $f_{34}(=f_{43})$ will all be zero.

F_1 and f_{41} are equal and opposite. The moment $F_1 \times a$ is balanced by M^1 . ($M^1 + F_1 \times a = 0$)

(b) F_2 alone acting: From the fbd of link 2- Forces F_2 , f_{32} (along link 3, being 2-force member) are collinear, which determines the direction of f_{12} (fig.c.2). Now complete the force polygon to determine the magnitudes of f_{12} and f_{32} as well. (fig.c.3). Also, $f_{23} = f_{43}$

On link 1, f_{41} and f_{21} are equal and opposite, and balanced by M^2 given by

$$M^2 + f_{21} \times b = 0.$$

© Force F_3 on Link 3 alone (Fig.c.4): Consider fbd of link 3. F_3 , f_{23} and f_{43} are collinear, from which directions of f_{23} and f_{43} are known. Their magnitudes are known from force polygon. f_{32} ($= f_{12}$) are the forces acting on link 2.

Forces on link 1 are f_{21} and f_{41} . are equal and opposite (Fig.c.5), and their couple is balanced by $M^3 (= f_{21} \times c)$.

The turning moment required under the simultaneous action of all forces is

$$T = M^1 + M^2 + M^3$$

Note: Each joint force is similarly obtained by superimposing the particular joint force Obtained in the 3 cases.

