Linearisation

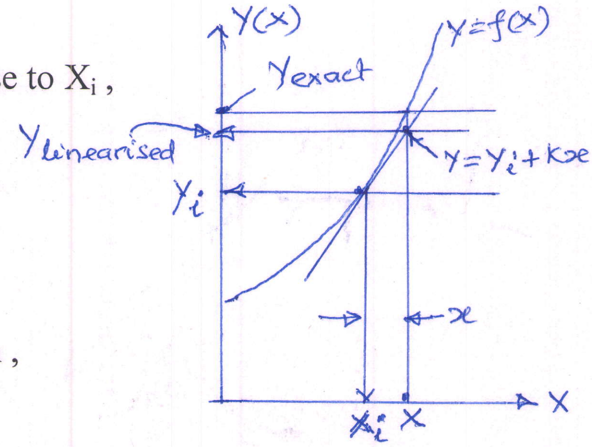
For Linear systems relations between variables are linear differential equations, usually with constant coefficients.

In reality, many control components have non-linear relations. These need to be converted or approximated as linear relations, which enables us to use linear control theory concepts. For this purpose, the slope at the operating point is used as a constant to give approximate linear relation.

Such linearization, though approximate, leads to acceptable mathematical models. Usage of such model does not affect the analysis or performance, particularly when feedback is used, as the output is fedback to modify the plant input so that output matches the reference input.

(i) Y = f(X) – function of one variable. (given as non-linear relation)

Let Y = f(X) be a non-linear equation (such as $y = 2X^2 + 10$ etc) represented graphically as shown in the figure.



If $Y = Y_i$ at $X = X_i$, then at any point X close to X_i,

given by $X - X_i = x$

$Y = Y_i + (dY/dX)_i\ x$

or $Y - Y_i = (dY/dX)_i\ x$

or $Y = Yi + K x$ is the linear approximation ,

K being the slope of Y at $X = X_i$.

<u>Example</u>: Take a non-linear relation $\mathbf{Y = X^2}$. If we know that the system operates around a point $X_i = 10$,

$Y = 100$, and $dY/dX = 2X = 20$;

Thus $Y = Y_i + (dY/dX)\ x = Y_i + 20\ (X - X_i) = 100 + 20(X-10)$

or **$Y = 20X - 100$ is the linearized equation.**

To see the error, suppose we want Y at X = 11. The exact value is $Y = 11^2 = 121$. The approximate value from above linear relation is $Y = 20 \times 11 - 100 = 120$.

The error is less than 1% and the linearised relation would be acceptable.

(ii) When Y is a function of many variables $Y = f(A, B, C)$

When the value of Y at any operating point (at values A_i, B_i and C_i) is Y_i, then change in Y for changes a, b and c from the operating point i, is given as

$y = (\partial Y/\partial A)\, a + (\partial Y/\partial B)\, b + (\partial Y/\partial C)\, c$.

As the slopes are constant at the operating point, we can write the change in Y as

$y = (Y - Y_i) = K_1 a + K_2 b + K_3 c$, as a linear relation.

Example: To find a linear relation for M, the mass flow of air through a restriction, given by $M = 0.04\, A\, P/\sqrt{T}$, where M = mass flow of air in kg/s, T is temp in Kelvin, P is pressure diff. across orifice in N/m^2 , A is area in m^2 . (at a constant T)

About an operating point, where $A = A_i$, $P = P_i$, and $T = T_i$ (when $M = M_i$).

$M - M_i = m = (\partial M/\partial A)\, a + (\partial M/\partial P)\, p$

a being the change from A_i and p the change from P_i. Thus

$m = (0.04\, P/\sqrt{T})\, a + (0.04\, A/\sqrt{T})\, p$

$\quad = (M_i/A_i)\, a + (M_i/P_i)\, p$

To get the linear relation at the operating point , $T_i = 300$ K, $P_i = 0.2 \times 10^6$ N/m^2 , $A_i = 10^{-4}$ m^2, which gives $M_i = 0.0462$ kg/s, substitution in the above equation gives,

$m = 462\, a + 0.231\times10^{-6}\, p$

Or $\mathbf{M = M_i + m = 0.0462 + 462\, a + 0.231\times10^{-6}\, p}$

This is the linearized relation for the mass flow rate about the operating point for changes in A and P.

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