Impulse & Momentum

For linear motion

$F = m.a = m\ dv/dt$

$\int F.dt = \int m.dv$

or $F(t_2 - t_1) = m(v_2 - v_1)$

The product of F and dt is called Force Impulseand has N.s as units.

When angular momentum is involved,

$M = I\ d\omega/dt$

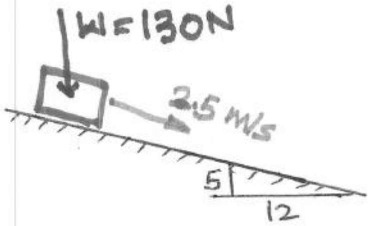
$\int M.dt = \int I.d\omega$

$M(t_2 - t_1) = I(\omega_2 - \omega_1)$

The product of M and dt is called Angular Impulse and has N.m.s as units.

<u>Net Impulse = Change in Momentum(linear or angular)</u>

***Example 1**: A block weighing 130 N starts moving down a slope with a velocity of 2.5 m/s. Find its velocity after 5 secs. μ =0.3*



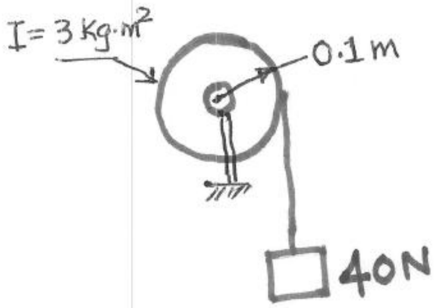
We may use F.t = m (V_2 –V_1);

F in the direction of motion = F Sin Θ - μ F Cos Θ

$= 130\text{x } 5/13 - 0.3 \text{ x } 12/13\text{x}120 = 50 - 36 = 14$ N

F.t =14 x 5 = m (V_2 –V_1) =(130/g) (V_2 – 2. 5) → **V_2 = 7.78 m/s.**

***Example2**: A block of 40 N connected to a pulley (I = 3 Kgm^2) is allowed to move down. Determine the ang .velocity of pulley after 5 sec. Bearing friction = 0.5 Nm*



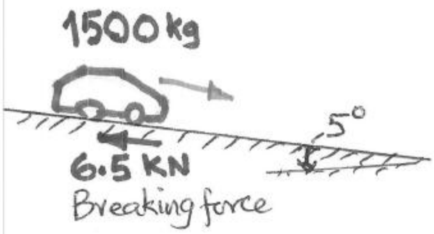
Net Angular impulse = M.dt

= (40x0.10 - 0.5)x 5= 17.5 Nm

Hence M.dt = Iω_2 – Iω_1→17.5 = 3ω_2

ω_2 = 5.67 rad/s

***Example 3:** A 1500 kg car moves down a 5^o slope at 100 kmph, when brakes are applied. The breaking force is constant at 6.5 KN. Find the time taken for the car to come to rest.*



Effective force on the car is its weight component minus the braking force. Hence

(W Sin5^o – 6500)t = mV_2-mV_1 = - m V_1

V_1 = 100 kmph = 100 x1000/3600 = 100/3.6 m/s

(1500g Sin5- 6500) t= 1500 x (100/3.6)→ **t = 10.08 s**

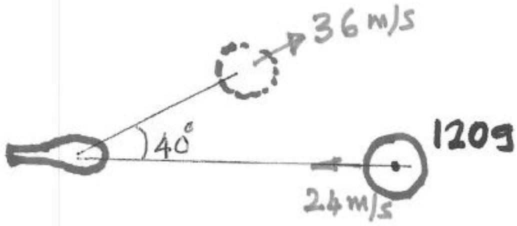
Example 4: A baseball 120 g is pitched with 24 m/s velocity and after hit has 36 m/s in the direction shown. If contact is 0.015 s, find average impulse force exerted on the ball.

In view of change in direction, Impulse force has also a component in perpendicular direction.

X drn. -- $F_X .t = m(V_{2x} - V_{1x})$

$F_X \times 0.015 = 0.12 (36 \cos 40 - (-24))$

$F_X = 412.6$ N



Y drn :$F_Y \times t = m(V_{y2} - V_{y1})$

$F_Y \times 0.015 = 0.12 \times 36 \sin 40$

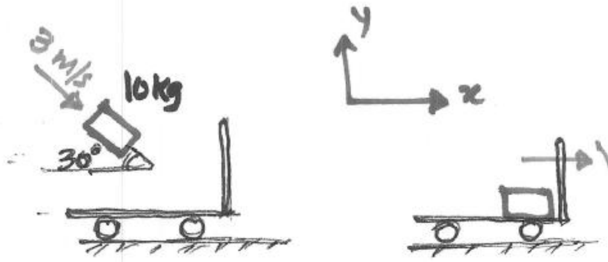
$F_y = 185.$ N

$F_t = 452 \angle 24.2^\circ$

Example4: *A 10 kg package at 3 m/s is thrown on a cart of 25 kg at 30° angle. Find (a) velocity of cart (b) Velocity impulse by cart on package (c) fraction of energy lost.*

$M_1 V_1 = (m_1 + m_2) V_2$

$10 \times 3 \cos 30 = (10+25) V_2 \rightarrow V_2 = 0.742$ m/s



(b) impulse on ~~cart~~ package is given by:

$M_p V_1 \cos 30 + F_x .\Delta t = m_p V_2$

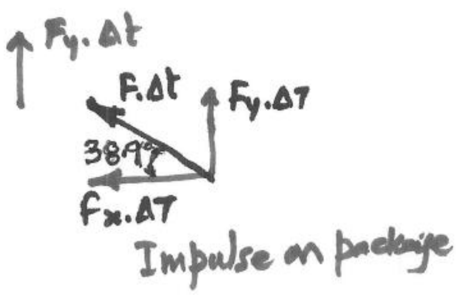
$10 \times 3 \cos 30 + F_x .\Delta t = 10 \times 0.742$

→ Impulse $F_x .\Delta t = -18.56$ N.s

Also along Y direction; $m_p V_1 \sin 30 + F_y .\Delta t = 0$

$10 \times -3 \times 0.5 + F_y .\Delta t = 0 \quad \rightarrow F_y .\Delta t = 15$ Ns

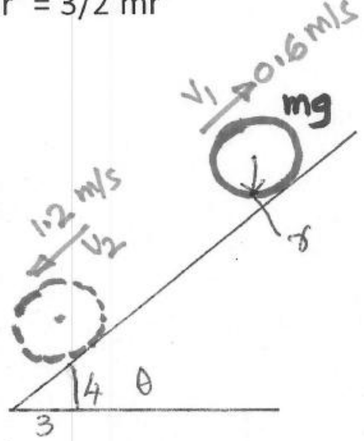
Impulse $F.\Delta t = 23.9 \angle -38.9^\circ$



(c) Energy lost = KE of pack – final KE of (cart +pack)

$= ½ \times 10 \times 3^2 - ½ (10+25) \times 0.742^2 = 45 - 9.635 = 35.37$ N.m

***Example 5:** A cylinder is given initial velocity of 0.6 m/s along a ~~down~~upward slope shown. Find the time required to attain 1.2 m/s.*



MI of cylinder about the periphery , $I = mr^2/2 + mr^2 = 3/2\ mr^2$

Force acting on cylinder = mg Sinθ x r

Angular Impulse = change in Ang. momentum

$(mg\ \sin\theta \times r) \times t = I\ (\omega_2 - \omega_1) = I\ (V_2/r - V_1/r)$

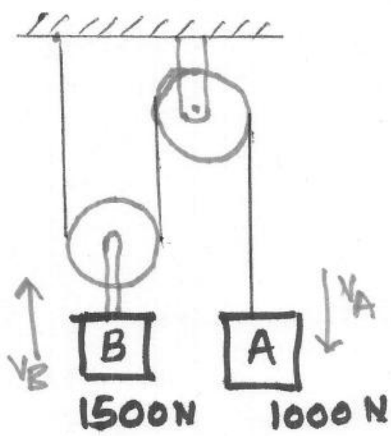
$= 3/2\ mr^2(V_2/r - V_1/r)$

→r gx4/5. r.t = 3/2. r(-1.2 - 0.6)

→ t = 5/4g x 3/2x1.8 = 0.35 s.

Time is a function of slope only.

***Example 6:** In the fig shown, if A & B are at rest initially, find the velocity of B, 10 s after starting from rest.*



We have $V_A = 2\ V_B$

For block A, the impulse equation is:

$(1000\text{-}T)t = m_A V_A = (1000/g)\ V_A = 2000/g.\ V_B$ .. (i)

For B, $(1500\text{-}2T)t = 1500/g.\ (-V_B)$

Or $(2T - 1500) = 1500/g.\ V_B$ .. (ii)

Eqns. 2 x(i) + (ii) → $(2000\text{-}1500)\times 10 = 5500/g.\ V_B$

→ $V_B = 8.92$ m/s

***Example 8**: When released from rest find (a) acceleration of blocks A & B;*
(ii) Tension in the chord.

Net force = 120g - μ 150g = $(M_A + M_B)$ a = (120+150) a
→ a = 3.27 m/s^2

(ii) Consider the FBD of 150 kg block,

T- μ 150g = 150 x a = 150x 3.27 → T = 785 N.

***Example** 9. A cylinder is pushed up with a velocity of 0.6 m/s, and rolls down at 1.2 m/s after time T. Find T given slope is 4/3; (Sinϑ =0.8)*

Torque x time = $I(\omega_2 - \omega_1) = I(V_2/r - V_1/r)$

$(mg\sin\theta \cdot r) \times T = (mr^2/2 + mr^2)(V_2/r - V_1/r) = 3/2\ m\ r(V_2 - V_1)$

$T = (3/2g \sin\theta)(V_2 - V_1) = (3/2 \times 9.81 \times 0.8)(1.2-(-0.6)) = 0.35$ s

(Note: The time taken is independent of the mass of cylinder)

Conservation of Momentum:

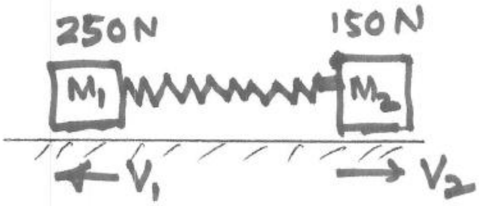
***Example** 1. Blocks A and B of 250 N and 150 N connected by spring and at 0.3 m distance are compressed to a distance of 0.15 m and released. Find the velocity of each when again at a distance of 0.3 m. K = 1200 N/m*

The potential energy of the spring is converted into the KE of both masses.

$\frac{1}{2} K \delta^2 = \frac{1}{2} M_1 V_1^{2+} + \frac{1}{2} M_2 V_2^2$

$\frac{1}{2}.1200.\ 0.15^2 = 1/2g.\ (250\ V_1^2 + 150\ V_2^2)$

$250\ V_1^2 + 150\ V_2^2 = 27\ g$ .. (1)



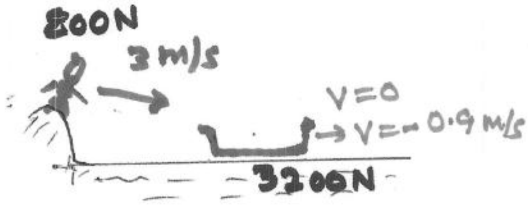
Conservation of Momentum: $-M_1V_1 = M_2 V_2$

$V_2 = 250/150\ V_1 = 5/3.V_1$

Which gives V_1 = 0.63 m/s; and V_2 = 1.05 m/s (from Eqn.1)

***Example**2: A 800 N man jumps of a pier horizontally with a velocity of 3 m/s into a 3200 n boat. Find boat velocity if (i) boat is at rest initially (ii) boat is traveling in opposite direction with 0.9 m/s.*

m1.V1 = (m1+m2)V



(i) 800/g. 3 = (800 +3200)/g. V
→ V = 0.6 m/s

(ii) 800/g. 3 + 3200/g. (-0.9) = (800 +3200)/g. V
→ V = -0.12 m/s

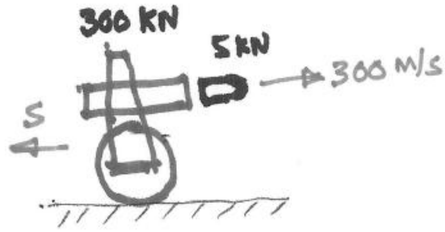
In case (ii) boat continues to travel towards the pier along with the man.

***Example**3: A gun weighing 300 KN fires a 5 KN projectile at 300 m/s. Find the velocity of gun recoil. (iii) if recoil is overcome with 600 KN, how far and how long will the gun travel.*

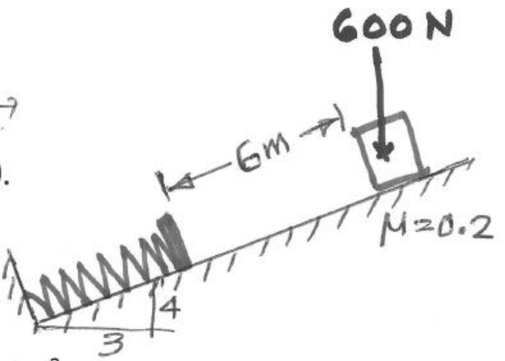
Cons. Of Momentum: 5x300 + 300xv =0 -> v = -5 m/s

Work Energy: 600 x s = ½ (300/g)x 5^2 -> s = 0.837 m

Impulse Momentum: 600 x t = 300/g.(0-5) -> t = 0.255 s



***Example**4: A block of 600 N slides on slope 4/3 and strikes a spring (k=200 N/m). Find the max. deformation of spring and maximum velocity*:



Work done = (600 sinθ - μ 600 cos θ) x (6 +d) = 1/2 .K. d^2

(600x4/5 - 0.2x600x3/5)(6+d) = ½.1200.d^2 →

⇨ →d = 2.37 m $d^2 - 0.67d - 4.02 = 0$ →

Velocity is maximum when acceleration =0 or Net force =0.

Ie.,(600 sinθ – μ 600 cos θ – K d_m) =0

Ie., 600x4/5 -0.2x600x3/5- 1200 d_m =0 ⇨d_m = 0.34 m

Work done = PE in spring compression + KE of block.

(600 sinθ - μ 600 cos θ) x (6 + 0.34) = ½.K.0.34^2 + ½. (600/g) V^2

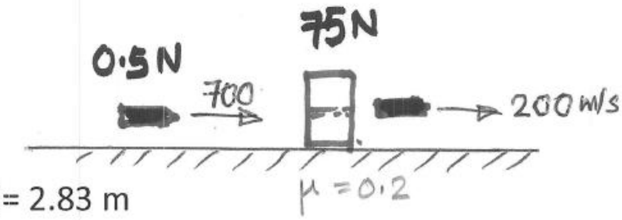
→V = 9.07 m/s. (Maximum Velocity)

***Example**5: A bullet of 0.5 N at 700 m/s strikes a block of 75 N at rest, pierces and travels at 200 m/s. Find velocity of block, distance travelled and time taken before coming to rest.*

Conservation of Momentum: m

$mU_B + M\,U_1 = m\,V_B + M\,V_2$

0.5/g. (700-200) = 75/g. V2 ⇨ v2 = 10/3 m/s



Work- Energy equation;

(a) F.d = ½ mv^2 ie., μ75d = ½ 75/g.$(10/3)^2$→ d= 2.83 m

Impulse: μF_f x t = M ($V_2 - V_1$) = M V_2

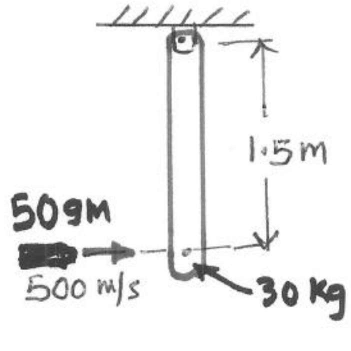
→ 0.2x75t = (75/g) x 10/3 → t = 1.7 s

Example *6: A bullet of 50 gm and at 500 m/s strikes a compound pendulum of 30 kg, and rad of gyr = 0.95 m. Find the angular vel. of compound pendulum along with embedded bullet.*

$I_{\text{plate+ bullet}} = m\, r_g^2 + m_b d^2$

$= 30 \times 0.95^2 + 0.05 \times 1.5^2$

$= 27.19\ \text{kg.m}^2$



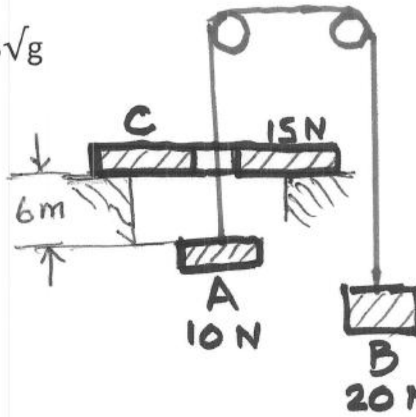
The moment of momentum of the bullet is then equal to the Angular Momentum of pendulum along with embedded bullet.
Hence $r \times m_1 V = I\,\omega$
$1.5 \times (0.05 \times 500) = 27.188\,\omega \rightarrow \omega = 1.38$ rad/s

Example*7.A moves 6 m before it picks C. How far does A move before reversing its direction. $W_A = 10$ N; $W_B = 20$ N, $W_C = 15$ N.*
Before impact: Work – Energy eqn: $(W_B - W_A) \times 6 = ½ (M_a + M_B) V_A^2$

Or $(20\text{-}10)\text{x}6 = ½. (20+10) V_A^2 \Rightarrow V_A = 2\sqrt{g}$

Velocity after impact :$(M_a + M_B)V_1 = (M_a + M_B + M_c)V_2$

Ie $V_2 = (20+10)/20+10+15) \times V_1 = 30/45 \times 2\sqrt{g} = 4/3\sqrt{g}$



Distance moved before (A+C) comes to rest

$(W_B - W_A - W_C) \times s = ½ (M_a + M_B + M_c) (0 - V_2^2)$

ie $5\,s = ½ \times 45 \times (4/3\sqrt{g})^2$; $\rightarrow$ **s = 8 m.**