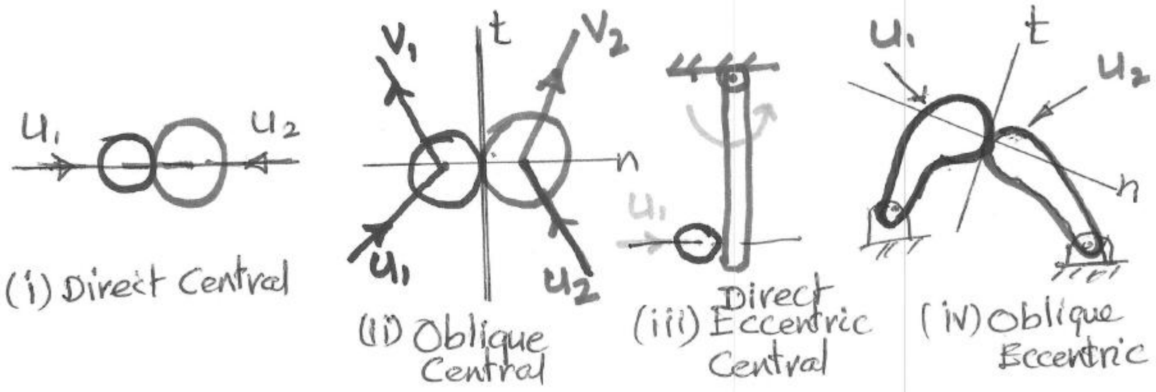
IMPACT

Impact of bodies involves exchange of energy and consequent changes in velocities. Some energy would usually be lost during impact.

Types of Impact: with respect to the line of impact and the velocities o impacting bodies, impact comes under any of the following types:

i. **Direct Central** : bodies move in the same line before and after impact
ii. **Oblique Central:** Velocities of bodies make an angle with the line of centres, called oblique angles.
iii. **Direct Eccentric:** Impact is eccentric to the centre of rotation of one body.
iv. **Oblique Eccentric:** As above at an oblique angle.

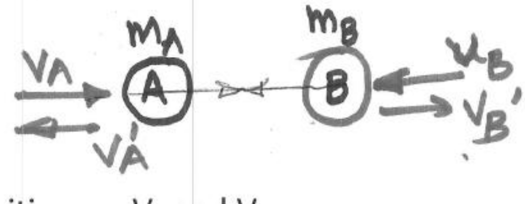


Coefficient of Restitution (CoR):

COR gives an idea of the energy lost in the process of impact of colliding bodies.

Consider two objects colliding in direct central impact.

Conservation of momentum gives

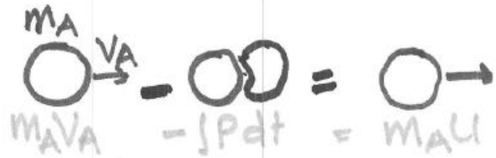


$m_AV_A + m_bV_B = m_AV_A' + m_bV_B'$ -- (1)

The three phases are before impact when velocities are V_A and V_B .

During impact for certain duration dt, the bodies are in deformation phase after which they stick together and travel with the same velocity U. After this phase is the phase of restitution when the bodies separate in the deformed shape or gaining original shape and travel with velocities V_A' and V_B'.

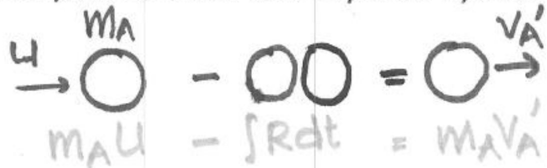
Considering body A of mass m_A, if the impulse force due to B on A is P, then the momentum after impact is given by



$m_A V_A - \int P\,dt = m_A U$ -- (2)

During the period of restitution, let the impulsive force on A by B be R, then

We have,



$m_A U - \int R\,dt = m_A V_A'$ -- (3)

The ratio of the Force impulsive during restitution to that during deformation is called the Coefficient of restitution and denoted by e.

Thus e from (2) and (3)

is $e = \frac{\int R\,dt}{\int P.dt} = (U - V_A')/(V_A - U)$

Similarly for B impacting A,

we get $e = (V_B' - U)/(U - V_B)$

We can write, $e = (U - V_A')/(V_A - U) = (V_B' - U)/(U - V_B)$

$= (U - V_A' + V_B' - U) / (V_A - U + U - V_B)$

Or $e = (V_B' - V_A')/(V_A - V_B)$ -- (4)

Or $(V_B' - V_A') = e\,(V_A - V_B)$

Rel. Velocity after impact = e. (Rel. Velocity before impact)

The velocities V_A' and V_B' can be obtained from equations (1) and (4)

<u>For e=0, is a case of perfectly Plastic impact</u>, and the two bodies stay together and have certain velocity, while momentum is conserved.

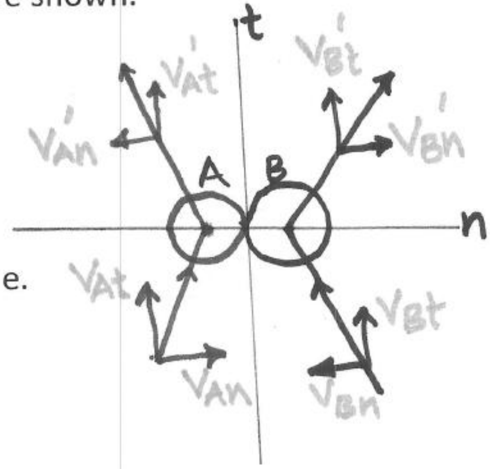
<u>e = 1 is a case of perfectly Elastic impact</u>, where relative velocities before and after are the same. In this case, it can be shown that the Kinetic Energy is conserved.

Oblique Central Impact:

In this case, the velocities of the objects are not along the line of impact (line of centres).

Direction and magnitude of the velocity after impact are unknown.

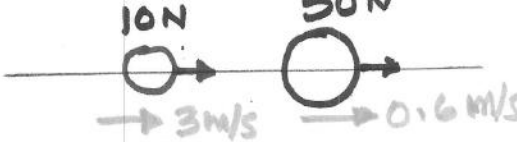
The components of velocity before and after impact, for both objects, along the line of impact n, and along common tangent line t are shown.



- The t components, (those along tangent line) are not affected and remain the same.
  $V_{At} = V'_{At}$ and $V_{Bt} = V'_{Bt}$
- The n components are affected by impact and the relative n component velocities are related by e.
  $V'_{Bn} - V'_{An} = e\,(V_{An} - V_{Bn})$ -- (1)
- Components of the total momentum along n is conserved.
  $m_A V_{An} + m_b V_{bn} = m_A V'_{An} + m_b V'_{bn}$ -- (2)
  Eqns. (1) and (2) give the n components after impact, and as t components are unchanged and known, velocities of both objects can be determined.

***Example 1**: Two elastic balls of 10N and 50 N travelling along a line in the same direction at 3 m/s and 0.6 m/s collide. Find the velocities after impact if e = 0.75.*

Conservation of Momentum: $10 \times 3 + 50 \times 0.6 = 10 \times V_1 + 50 \times V_2$ ---- (i)



CoR = $e = 0.75 = (V_2 - V_1) / (U_1 - U_2)$

$(V_2 - V_1) = 0.75 \times (3 - 0.6) = 1.8$ -- (ii)

Eqns. (i) and (ii) ⇨ $V_1 = -0.5$ m/s $V_2 = 1.3$ m/s;

Loss of Kinetic Energy = $(\frac{1}{2} M_1 U_1^2 + \frac{1}{2} M_2 U_2^2) - (\frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2)$

= 1.07 Joules

Example 2: *For e= 0.9 Find V_1 and V_2.* $(U_1 = 9\ m/s;\ U_2 = 12\ m/s;\ m_A = m_B = m)$

Components of velocity along normal to line of impact remain the same

IeU$_{Ay}$= V$_{Ay}$⇨ 9 Sin 30 = 4.5 m/s. U$_{By}$= V$_{By}$= 12 Sin 60 = 10.39 m/s

(i) $e = \frac{(V2 - V1)}{(U1 - U2)} = \frac{Vbx - Vax}{Uax - Ubx}$

$= \frac{Vbx - Vax}{(\cos 30 + 12\ Cos\ 60} = 0.9$

(ii) conservation of momentum gives

m.Uax + m.Ubx = m.Vax + m.Vbx, ie

m.9Cos 30 – m.12 cos 60 = m. (Vax + Vbx)

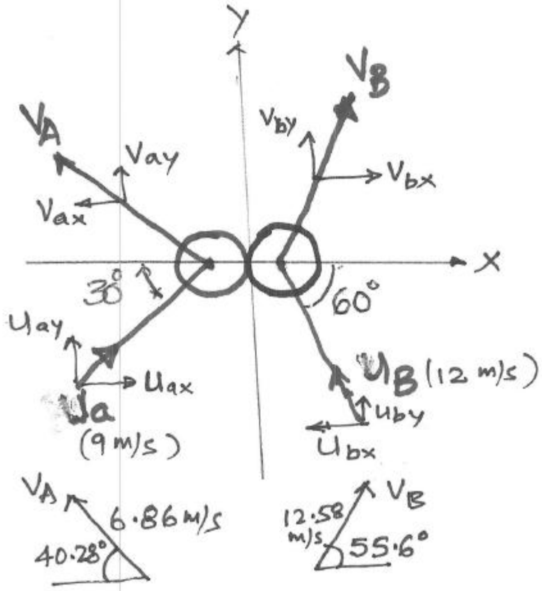
Vax + Vbx = 7.794 – 6 =1.794 ---(2)

From (1) and (2), Vax = -5.31 m/s; Vbx = 7.104 m/s.

Va = $\sqrt{(Vax^2 + Vay^2)} = \sqrt{(5.31^2 + 4.5^2)}$= 6.86 m/s

$\theta_A = \tan^{-1}(4.5/5.31) = 40.28^\circ$

Similarly, $V_B = 12.58$, $\angle 55.6$



Example 3*: A ball is thrown at an angle θ to vertical and rebounds at Φ. Show that e = tan θ/ tan Φ .*

e = rel. vel. After impact /rel. vel before impact .

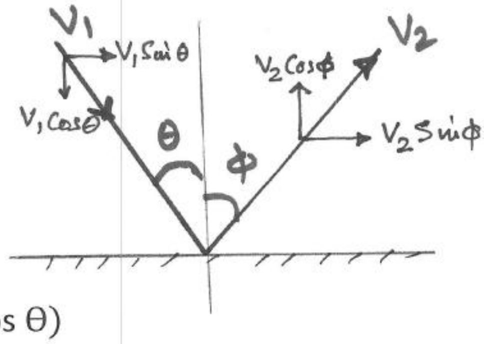
Vel. Of floor is zero. Also, the horizontal velocity, along the floor is not affected.

Hence $V_1 \sin\theta = V_2 \sin\Phi$. ——— (1)

$e = V_2 \cos\Phi / V_1 \cos\theta = (\sin\theta/\sin\Phi).(\cos\Phi/\cos\theta)$

from (1)

$= \tan\theta/\tan\Phi$

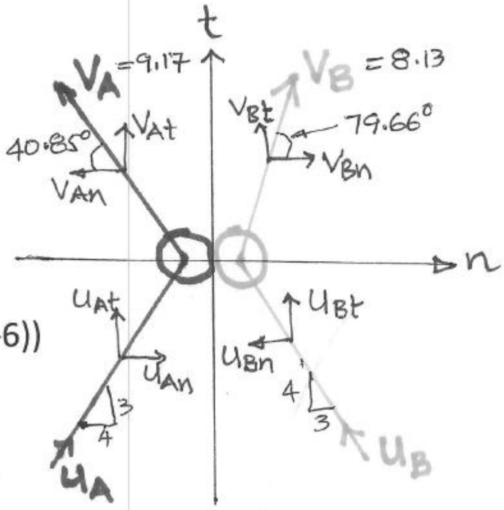


$$e = \frac{V_{2(floor)} - V_{2(ball)}}{V_{1}(ball) - V_1(floor)} = \frac{0 - V_2\cos\Phi}{-V_1\cos\theta - 0} = \frac{V_2\cos\Phi}{V_1\cos\theta}$$

Example 4: *For two discs on a plane for oblique central impact shown, e = 0.6 find velocity after impact. Given $W_A = 10$ N; $W_B = 20$ N; $U_A = U_B = 10$ m/s*

The tangential velocities (along t) are the same before and after impact.

$U_{At} = V_{At} = 6$ m/s; $U_{Bt} = V_{Bt} = 8$ m/s



In the normal direction, from Cons. of momentum,

$W_A U_{An}/g + W_B U_{Bn}/g = W_A V_{An}/g + W_B V_{Bn}/g$

or $10 \times 8 + 20 \times (-6) = 10 \times V_{An} + 20 \times V_{Bn}$

$\Rightarrow V_{An} + 2V_{Bn} = -4$ --- (i)

$e = 0.6 = (V_{Bn} - V_{An}) / (U_{An} - U_{Bn}) = (V_{Bn} - V_{An}) / (8-(-6))$

$(V_{Bn} - V_{An}) = 8.4$ --- (ii)

From (i) and (ii), $V_{An} = -6.94$ m/s; $V_{Bn} = 1.46$ m/s.

Resultant Velocity of A after impact, $V_A = \sqrt{(V_{An}^2 + V_{at}^2)}$

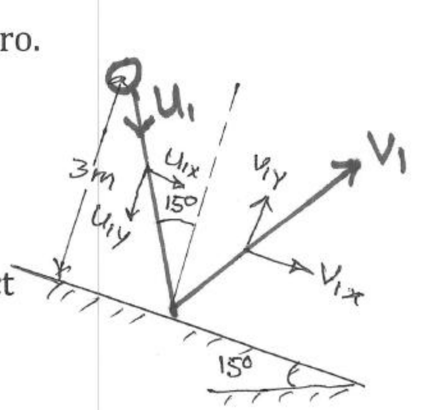
$\mathbf{V_A} = \sqrt{(6.94^2 + 6^2)} = \mathbf{9.17}$ **m/s** at $\Theta_A =$ $\mathbf{\tan^{-1}(V_{At}/V_{An}) = \tan^{-1}(6/6.94) = 40.85^\circ}$

Similarly $\mathbf{V_B} = \sqrt{(1.46^2 + 8^2)} = \mathbf{8.13}$ **m/s** at $\mathbf{\Theta_B = \tan^{-1}(8/1.46) = 79.66^\circ}$

Example 5: *A ball is dropped on a floor at 15° inclination from a height of 3 m. If the coefficient of restitution is e = 0.8, find velocity after impact.*

Let U_1, V_1 be the velocities of the ball before and after impact.

And U_2 and V_2 be that of ground, which are both zero.



$U_1 = \sqrt{2gh} = \sqrt{2gx3} = 7.67$ m/s

$U_{1x} = V_{1x} = U_1 \sin 15 = 1.986$ m/s

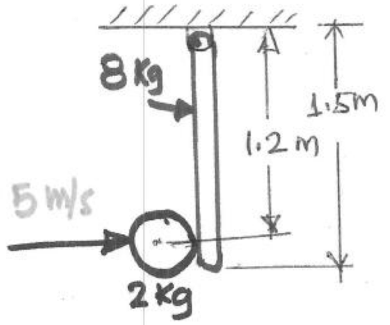
Component along normal only is affected by impact and is $U_{1y} = 7.67 \cos 15 = 7.412$ m/s.

$E = (V_2 - V_1) / (U_1 - U_2) = (0 - V_{1y}) / (U_{1y} - 0)$

$0.8 = (0 - V_{1y}) / (-7.412 - 0) \rightarrow V_{1y} = 5.929$ m/s.

$V_1 = (V_{1x}^2 + V_{1y}^2) = 6.253 \angle 71.5^\circ$

***Example 6**: A ball of 2 kg travelling at 5 m/s strikes a hinged plate of 8 kg at a distance of 1.2 m from the hinge. If CoR is 0.7, find the angularvelocity of the plate.* (soon after impact)



$I = ml^2/3 = 8 \times 1.5^2/3 = 5.227 \text{ Kg.m}^2$

Conservation of Angular momentum:

$M_bU_b = M_b.V_b x 1.2 + I\omega$

$2x5x1.2 = 2xV_b x1.2 + 5.227\ \omega$

$12 = 2.4\ V_b + 5.227\ \omega$ -- (i)

$CoR = e = 0.7 = (V_p - V_b)/(U_b - U_p) = (1.2\ \omega - V_b)/(5-0)$

$1.2\ \omega - V_b = 3.5$ --- (ii)

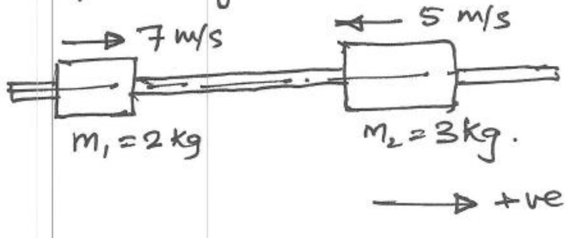
Eqns. (i) and (ii)$\rightarrow \omega = 2.52$ rad/s; $V_b = -0.48$ m/s.

The ball reverses in direction after impact.

IMPACT OF BODIES — EXAMPLES

Ex. 1: Note: Objects impacting are numbered as 1 and 2.
Velocity before impact is u and after impact is V.
u_1 and V_1 are the velocities of body 1.

Example-1: Two cylinders sliding smoothly on a rod as shown collide. Find velocities after impact if $e = 0.6$.



From Conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$
$$2\times7 + 3\times(-5) = 2V_1 + 3V_2$$
$$\text{or } 2V_1 + 3V_2 = -1 \quad \text{--- (1)}$$

Also $$e = 0.6 = \frac{V_2 - V_1}{u_1 - u_2} = \frac{V_2 - V_1}{7 - (-5)} = \frac{V_2 - V_1}{12}$$

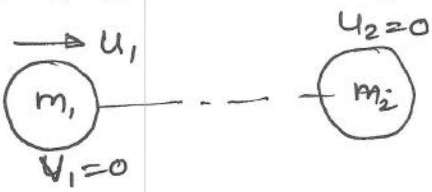
$$V_2 - V_1 = 7.2 \quad \text{—— (2)}$$

Eq. (2) in (1) gives $2V_1 + 3(V_1 + 7.2) = -1$

$5V_1 = -22.6$ or $\underline{V_1 = -4.52}$ m/s

From (2) $\underline{V_2} = V_1 + 7.2 = \underline{2.68}$ m/s

Ex. 2: For two balls impacting as shown, what should be the ratio of masses if COR = e. (m_2 at rest initially), so that m_1 comes to rest after impact.



$$e = \frac{V_2 - V_1}{u_1 - u_2} = \frac{V_2 - 0}{u_1 - 0} = \frac{V_2}{u_1} \quad \text{—(1)}$$

Momentum equation is:

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$
$$\text{or } m_1u_1 + 0 = 0 + m_2V_2$$
$$\text{or } m_1u_1 = m_2V_2 = m_2(e \times u_1) \quad \text{from (1)}$$
$$\text{or } m_1u_1 = e\cdot m_2\cdot u$$

Hence $e = \dfrac{m_1}{m_2}$ for m_1 to come to rest after impact.

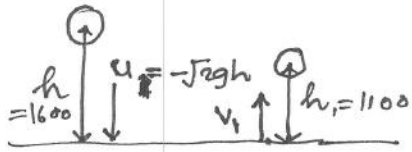
Ex. 3 A tennis ball is accepted if it rebounds to waist height (1100 mm) when dropped from shoulder ht (1600 mm). Then, what should be the value of 'e' and what is the percentage of energy lost in a bounce.

[Note: In problems where an object hits or impacts with floor, body 2 (floor) has $u = v = 0$.
Thus e becomes, $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v_1}{u_1 - 0} = -\frac{v_1}{u_1}$

Velocity of ball when dropped from ht. $h = \sqrt{2gh}$

$u_1 = \sqrt{2gh}$ (-ve); $v_1 = \sqrt{2gh_1}$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - (-\sqrt{2gh_1})}{\sqrt{2gh} - 0}$$



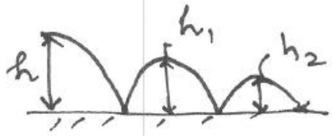
or $e = \frac{\sqrt{h_1}}{\sqrt{h}} = \frac{\sqrt{1100}}{\sqrt{1600}} = 0.829.$

$$\% \text{ Energy Lost} = \frac{\frac{1}{2}mu_1^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mu_1^2} = \frac{mgh - mgh_1}{mgh} =$$

$$= \frac{1600 - 1100}{1600} \times 100 = \underline{31.25\%}$$

Ex. 4 A ball is dropped from a height 'h'. If ht. of second bounce is h_2, find $\frac{h_2}{h}$ if COR = e.

[The horizontal component of velocity of ball remains the same and need not be considered. COR applies only to normal (or vertical) velocity.]



<u>For first bounce.</u>

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - (-\sqrt{2gh_1})}{\sqrt{2gh} - 0} = \frac{\sqrt{h_1}}{\sqrt{h}} = \sqrt{h_1/h}$$

or $\frac{h_1}{h} = e^2$

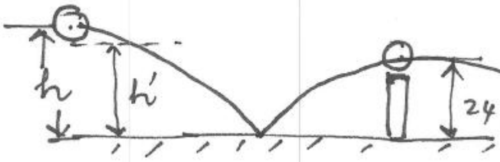
$|||^{ly}$ for second bounce, $\frac{h_2}{h_1} = e^2$

$$\frac{h_2}{h} = \frac{h_2}{h_1} \times \frac{h_1}{h} = e^4; \quad \boxed{\frac{h_2}{h} = e^4}$$

If ht. h_2 is given (after second bounce), we find $e = \left(\frac{h_2}{h_1}\right)^{1/4}$

Ex. 5 A table tennis ball just clears the net reaching 240 mm, after first bounce. From what height it should be hit horizontally? Radius of ball = 20 mm. $e = 0.9$.

As earlier,

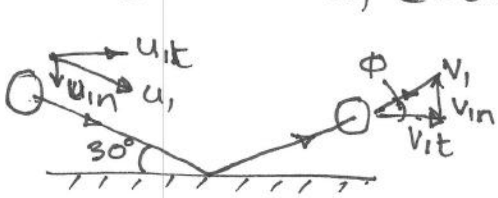


$$e = \frac{0-(-\sqrt{2gh_1})}{\sqrt{2gh}-0} = \frac{\sqrt{h_1}}{\sqrt{h}}$$

or $0.9 = \sqrt{240}/\sqrt{h}$

$$h = \left(\frac{\sqrt{240}}{0.9}\right)^2 = 296.3 \text{ mm.} \quad (\& \text{ Ht. } h' = 296.3 - r = \underline{276.3 \text{ mm}})$$

Ex. 6 A ball hits floor with 20 m/s velocity at 30°. If $e = 0.6$, find velocity after impact.



$u_{1t} = V_{1t} = 20\cos 30° = 17.32$ m/s.

$u_2 = V_2 = 0$ (of floor)

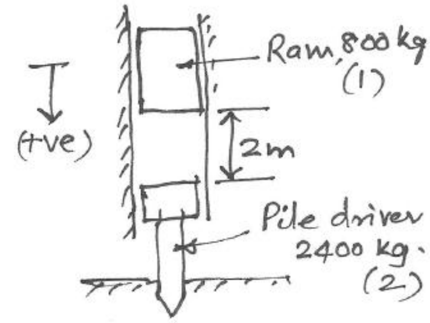
$$e = 0.6 = \frac{V_{2n}-V_{1n}}{u_{1n}-u_{2n}} = \frac{0-V_{1n}}{(-20\sin 30)-0} = \frac{V_{1n}}{10}$$

$\therefore V_{1n} = 0.6 \times 10 = 6$ m/s

$\therefore V_1 = \sqrt{V_{1n}^2 + V_{1t}^2} = \sqrt{6^2 + 17.32^2} = \underline{18.33 \text{ m/s}}$

$\& \ \phi = \tan^{-1}(V_{1n}/V_{1t}) = \tan^{-1}(6/17.32) = \underline{19.1°}$

Ex. 7 A 800 kg ram drops 2 m to hit a pile driver of 2400 kg. If ram rebounds up 0.1 m after impact, find
(a) Velocity of pile after impact (b) e (COR) and (c) Energy Lost



$u_1, V_1 \rightarrow$ Ram ; $u_2, V_2 \rightarrow$ driver

$u_1 = \sqrt{2gh} = \sqrt{2g \times 2} = 6.26$ m/s

$V_1 = \sqrt{2gh} = \sqrt{2g \times 0.1} = 1.4$ m/s. (-ve)

From Cons. of momentum: $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

or $800 \times 6.26 + 2400 \times 0 = 800 \times (-1.4) + 2400 \times V_2$

(a) $\rightarrow V_2 = 2.55$ m/s.

(b) $$e = \frac{V_2 - V_1}{u_1 - u_2} = \frac{2.55-(-1.4)}{6.26-0} = \underline{0.63}$$

(c) Energy Lost = Energy before Impact − Energy after impact.

$= \frac{1}{2}m_1u_1^2 - \left(\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2\right)$ $(mgh = \frac{1}{2}m_1u_1^2)$

$= 15700 - 8620 = \underline{7080 \text{ J}}$