Dynamic Force Analysis: Analytical Method

Let us consider a slider crank mechanism, and the forces acting on a position and the corresponding torques on the crank. In this part, we will look at the following aspects.

Acceleration of piston, and the inertia force caused, the force transmitted to crank pin through connecting rod and the torque on crank (or crank shaft). - In that considering the mass of the crank shaft.
The above – considering the mass of C.R in which the mass is distributed at piston and crank ends, and correction required thereby.
Difference in analysis between horizontal and vertical motion of piston (to see how granty affects).

Dynamic Force Analysis – Analytical Method Slider Crank Mechanism

Consider the figure, Let l/r = n

Also l Sin $\emptyset$= r Sin θ or Sin $\emptyset$ = r Sin θ/ l = Sin θ/n

Hence Cos $\emptyset$ = $(1- \sin^2 \theta/n^2)^{1/2}$ -----(1)

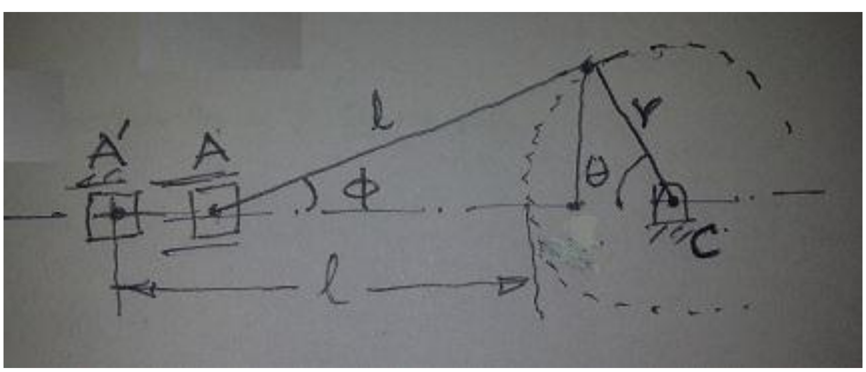
= $1- \sin^2 \theta/2n^2$ (from binominal expansion)

Now, displacement of piston, from inner dead centre when crank angle is θ, becomes

x = $\grave{A}$C – AC = l+r – (l Cos $\emptyset$ + r Cos θ)

= r(1- Cos θ) + l(1- Cos $\emptyset$)

= r(1- Cos θ + n(1- Cos θ)) -----(2)



However, from Eqn.(1), 1- Cos θ = $\sin^2 \theta/2n^2$

Substitution in Eqn(2) gives,

x = r(1- Cos θ + $\sin^2 \theta/2n$)

Velocity of piston = ${dx}/{dt}$ =(${dx}/{d\theta}$) . (${d\theta}/{dt}$) = $\omega \frac{dx}{dt}$

or $V_p = \dot{x} = \omega$.r (Sin $\theta + 2Sin\ \theta\ Cos\ \theta/2n$)

or $V_p = \omega$.r (Sin $\theta\ +\ Sin\ 2\theta/2n$) ----(3)

Acceleration of piston = $\ddot{x} = {dv_p}/{dt} = ({dv_p}/{d\theta}) \cdot ({d\theta}/{dt})$

or $a_p = \dot{x} = \omega^2$r (Cos $\theta + \frac{\cos 2\theta}{n}$) ----(4)

We also need to find an expression for Angular Acceleration of the connecting rod, which has an associated torque, when its mass is not negligible. (i.e. M.I. is not negligible).

We have Sin $\emptyset = Sin\ \theta/n$

or Cos $\emptyset \frac{d\emptyset}{dt} = (Cos\ \theta/n)\frac{d\theta}{dt}$

$\omega_c = \frac{d\emptyset}{dt} = \frac{\omega}{n}\frac{Cos\ \theta}{Cos\ \emptyset}$

$= \frac{\omega}{n}\frac{Cos\ \theta}{\sqrt{(1-\sin^2\theta/n^2)}}$

or $\omega_c = \omega\frac{\cos\theta}{n}$ (as n>> sin θ)

Angular Acceleration of C.R, - ${d\omega_c}/{dt} = ({d\omega_c}/{d\theta}) \cdot ({d\theta}/{dt})$

$\alpha_c = \frac{-\omega^2 sin\theta}{n}$ ----(5)

Consider an example, in which mass of CR is neglected.

Example 1: A steam engine of 200 mm bore, stroke = 300 mm, CR length = 625mm Mass of reciprocating parts = 15kg; speed = 250rpn; differential steam pressure = 840 kN/m^2, find (a) Crank torque (b) force on crank shaft bearings

From the figure

F_c = Force along CR = F_p/ Cos $\emptyset$

F_T = Force perpendicular to crank = F_c Cos $(90-\theta-\emptyset)$ = F_c Sin$(\theta+\emptyset)$

Effective crank torque = F_T x r

Force on crank shaft bearing = force along crank

= F_c Cos$(\theta+\emptyset)$

ω = 200 N/60 = 26.18 r/s; r = 300/2 = 150 mm = 0.15 M

$n = \frac{l}{r} = \frac{0.625}{0.15} = 4.167$

Net force on piston = $(P_1-P_2)A$

= 840 x 10^3 x $\frac{\pi}{4}(0.2)^2$ = 26390N

Piston Effort (F_p) = Force on piston – Inertia force due to piston near

= $(P_1-P_2)A$ - $m_p\omega^2$r (Cos θ + Cos20/n)

= 26390 – 15 x$(26.18)^2$ x 0.15 (cos 30 + Cos60/4.167)

= 26870 N

$Sin\,\emptyset$ = Sinθ/n = Sin30/4.167 $\Longrightarrow$ $\emptyset$ = 6.89^0

Force along CR =F_c = F_P/ Cos$\emptyset$

Crank Torque = r =(F_p/Cos$\emptyset$) Sin(θ + $\emptyset$).r

= (26870/ Cos 6.89^0) Sin(30+6.89^0) x 0.15 = = 2255 N.m

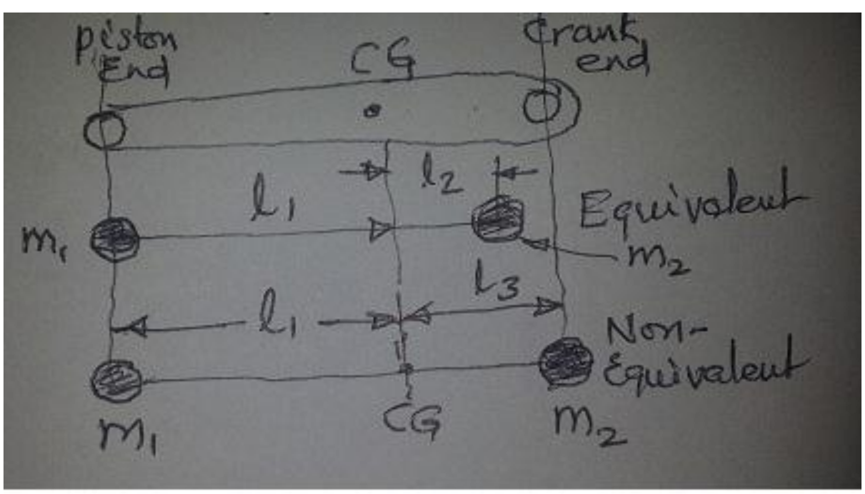
Force on Crank Shaft bearing = F_c cos (θ + $\emptyset$) = <u>20000 N</u>

<u>Dynamically Equivalent System:</u>

When the mass of connecting rod (CR) is not negligible, it has a moment of Inertia, and an associated torque when the CR is subjected to angular acceleration. The CR also has a linear acceleration which needs a force to overcome acceleration.

To simplify analysis, we take CR as a system consisting of mass m_1 at piston end and m_2 at crank end.

Such system wil be dynamically equivalent to CR when



(1) $m_1 + m_2$ = m (mass of CR)
(2) CG of both are at the same point ($m_1l_1 = m_2l_2$)

(3) M.I of two masses about CG is equal to that of CR.

i.e. $m_1 l_1^2 + m_2 l_2^2 = mk^2$

In figure (a), when l_2 is not specified, we can have an equivalent system where

$m_1 = \frac{l_2}{l_1 l_2}$ m; $m_2 = \frac{l_1}{l_1 l_2}$ m

However, if masses are placed at piston and crank (at lengths l_1 and l_3 from CG), we do not get a dynamically equivalent system,

M.I of the dyn. Non-equivalent system = mk_1^2, where $k_1^2 = l_1.l_3$

Hence the correction torque = $I_1\alpha - I\alpha$

$= m\, k_1^2\alpha - m\, k^2\alpha$

i.e $T_c = m\alpha(l_1 l_3 - l_1 l_2)$

The effect of the correction torque T_c (which is in the same direction as α_c) is a couple, the two forces acting at the ends of CR.

Example 2: Consider a problem of horizontal engine with data: stroke = 200 mm; CR length = 400 mm; CR mass = 100 kg. Mass of piston = 125 kg. Distance of CG from big end = 160mm. Radious of gyration of CR = 120 mm. when the crank angle $\theta = 30^0$

R= l/f = 400/100 = 4

We have $m_1 = m\, x(l_3/l) = 100\, x0.16/0.4 = 40$ kg

$m_2 = m\, x(l_1/l) = 100\, x0.24/0.4 = 60$ kg

Total mass at piston end = 125+40 =165 kg

$\omega = 2\pi \times 750/60 = 78.54$ r/s

Inertia force due to total mass at piston

i.e $F_p = (m_p+m_1)\, \omega^2 r\, (\cos\theta + \cos 2\theta/n)$

$= 165\, x(78.54)^2 \times 0.1\, (\cos 30 + \cos 60/4) = 6144.8$ N (to left)

T_1 = Torque on crank = $(F_p/\cos \emptyset)\sin(\theta + \emptyset).r$

$= (6144.8/\cos 7.18)\sin 37.18^0 \times 0.1 = 6145$ N.m (ccw)

T_c = Correction Torque = $m(k_1^2 - k^2)\, \alpha = m\, (l_1 l_3 - k^2)\, (-\omega^2\, \mathrm{Sing}\theta/n)$

$= 100(0.24 \times 0.16 - 0.12^2)\, (78.54^2\, \mathrm{Sin}30/4)$

= -1851 N.m (ccw)

<u>Note</u>: $-\alpha$ negative value indicates direction of reducing $\emptyset$

- Inertia torque is counter clock wise, as it is opposite to α
- Correction torque is clockwise, as it is opposite to Inertia torque.

T_2 = Torque on crank due to correction couple

$= (T_c/l \cos \emptyset) \times r \cos \theta = -1851 \times 0.1 \times \cos 30^0 / 0.4 \times \cos 7.18^0 = 404$ N.m

T_3 = Torque due to weight of $m_2 = m_2 g \times r \cos\theta = 60 \times 9.81 \times 0.1 \cos 30$

= 51 N.m (ccw)

Total Torque on crank shaft = 6145 + 1851 + 51 = 8047 N.m

For a vertical Engine:

- Subtract $(m_p+m_1)g$ from inertia force F_p
- T_3 instead will be $m_2 g\ r \sin \theta$ (clockwise)