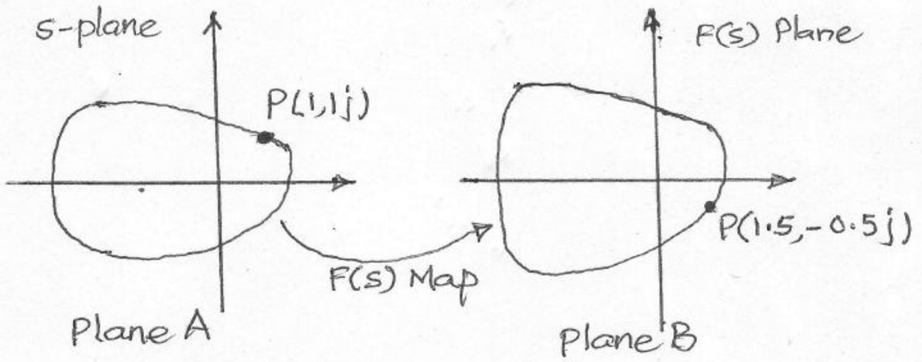
Nyquist's Stability Criterion

- Nyquist stability criterion is a frequency response technique used for determining the stability of a closed loop system, as well as the margins of stability.
- It uses complex mapping of the right half plane by the open loop transfer function G(s)H(s) to check whether the roots of the characteristic equation (C.E.), 1+ G(s)H(s) = 0, lie in the right half plane.
- Thus, we can determine whether a system is stable or not, since the roots of C.E. lying in the right half plane (or roots having +ve real parts) make the system unstable.
- From Bode or polar plot of G(s)H(s), we also determine Gain and Phase margins, the margins of stability that indicate how close the system is to unstable state.

Complex Mapping: Let us look at mapping of a closed path in a complex plane A to complex plane B, using a function F(s) = 5/(s+2).
Consider a closed contour C in plane A. Let a point P in plane A be (1, 1j).
Then F(s) = 5/(s+2) = 5/(1+1j +2) = 5/ (3 +j) = 5(3-j)/(9+1) = 1.5 – 0.5 j
Hence (1,1j) in A maps as (1.5 – 0.5 j) in complex plane B.



Each point s in plane A (called s-plane) is mapped to a point in plane B (called the F(s) map in this case). We map each point on A, moving in CW or CCW direction, to points in B. The path in B will also be closed.

Effect of location of poles and zeros in mapping:

Figure shows a complex plane in which a closed path is represented. Considering a function F(s) = (s-r), figure shows vectors from the origin to point **r** and to a point **s**. Vector **(s-r)** represents the term (s-r) . It can be seen that as **s** traverses the complete path, vector **(s-r)** rotates through one complete revolution. The angle of the function increases by 2π. If F(s) = 1/(s-r) instead, the angle decreases by 2π.

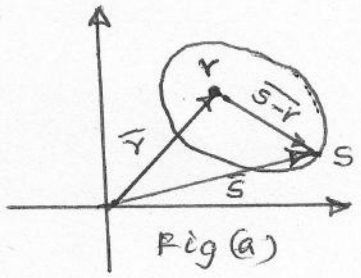


Fig (a)

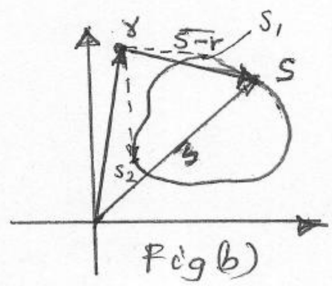


Fig b)

Fig (b) shows the case when **r** is outside the closed path. It can be seen that as **s** traverses the closed path, vector **(s-r)** moves to change the angle in one direction as s traverses from S_1 to S_2, while the angle changes by the same extent in opposite direction as s travels from s_2 to s_1. Thus the net change in the angle is zero.
Thus r inside the closed path increases the angle of the function F(s) = (s-r) by 2π while it makes no change if it lies outside. Also, if the term is in the denominator, the change in the angle is -2π.

Now for a function, F(s) $\frac{(s-Z1)(s-Z2)..(s-Zm)}{(s-P1)(s-P2)...(s-Pn)}$,

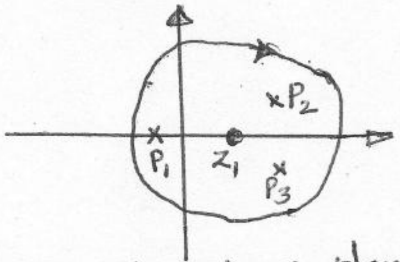
For each zero Zi, (root of numerator term), lying in the closed path s, the angle increases by 2π, and for each pole within the S contour, the angle decreases by 2π. Thus, it can be seen that F(s) map of the contour C in plane B will encircle the origin of B, N times given by

N = Nz – Np, where

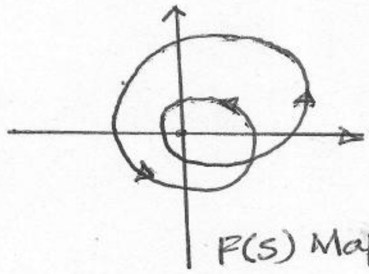
Nz = Number of zeros roots of Numerator of F(s) which are within contour C (called the zeros), and Np are the number of roots of the Denominator (called poles) lying within C.

Example: Let $F(s) = \frac{K\,(s+Z1)(s+Z2)..(s+Zm)}{(s+P1)(s+P2)..(s+Pn)}$

Take a contour C taken clockwise as shown. Suppose one zero, Z1, and 3 poles, P1, P2 , P3 are within C. Then, F(s) map of C in plane B will be such that it will encircle the origin N times, where N = Nz- Np = 1-3 = -2.
That means the F (s) map will encircle the origin 2 times CCW as shown.

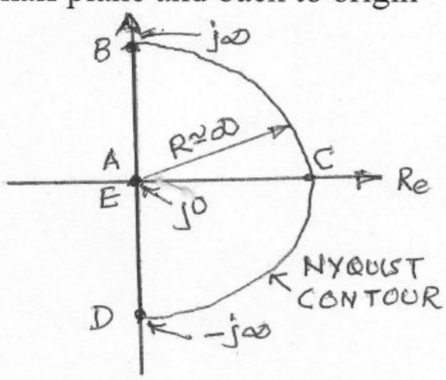


Contour in S-plane


F(S) Map of Contour

Nyquist Contour:

Let the C contour be taken as A-B-C-D-E, starting from origin of A, along Imaginary axis and sweeping infinite radius in right half plane and back to origin as shown. This is called the ***Nyquist Contour***.



Now let F(s) = 1+ G(s) H(s), the char. function.
The 1+ GH map of the Nyquist contour will encircle the origin N times, given by
N = Nz – Np, where
Nz = number of Zeros (or roots of the C.E., 1 + GH =0) lying within the Nyquist contour,
Np = number of Poles or roots of the denominator or poles of GH (or 1+GH) lying within the Nyquist contour.

However, Nz are the roots of the characteristic equation lying in the right half plane (within Nyquist contour), and those that make the system unstable. Hence Nz should be zero, for stability.

or Nz= N+ Np = 0, for stability.

or N = - Np

That is, the closed loop system will be stable if the 1 + GH map encircles the origin Np times in the counter clockwise sense.

Next, N is the encirclements of origin of 1+ GH map.

At the origin 1+GH = 0, or GH = - 1.

Hence, N, the encirclements of (0,0) of 1+GH map is the same as the encirclements of the point (-1,0) by GH map.

Since GH is more readily available in factored form, mapping is done with GH and N is the encirclements of (-1, 0) of GH map.

The statement for stability becomes:

Closed loop system will be stable if GH map will encircle the point (-1,0), Np times in the counter clockwise direction.

Again, *for an open-loop stable system*, all poles lie in left half plane, ie., they donot lie not within Nyqust contour. Hence Np=0. Then the stability equation becoms N = 0. Hence, for open-loop stable systems, the stability statement is:

For an open- loop stable system, the closed-loop system will be stable, if the net number of encirclements of (-1,0) by the GH map of the Nyquist Contour, is zero

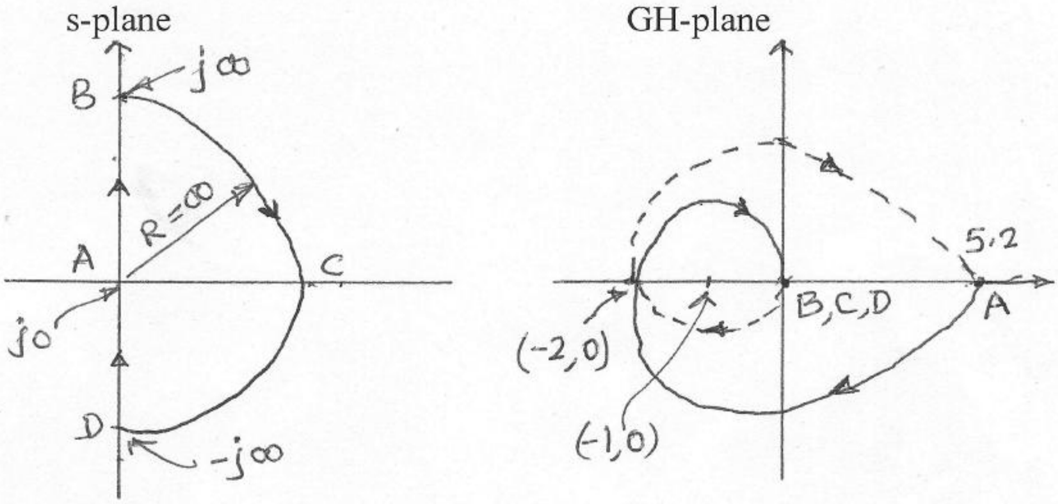
We will check the stability of some systems using Nyquist Criterion.

Example 1: Check this stability of a system which has

$$G(s)H(s) = \frac{52}{(s+2)(s^2+2s+5)} \text{using Nyquist Criterion.}$$

Draw a Nyquist Contour in the s-plane covering the whole right half plane.

Over A to B, ω varies from j0 to j∞, ie it covers the entire +ve imaginary axis. This map is the same as drawing the polar plot, shown as AB, in the GH-plane.



- Over B to D, let a point be

$S = R\, e^{j\beta}$ where β varies from + 90° to 0 to – 90°

$$G(j\omega)H(j\omega) = \frac{52}{(Re^{j\beta}+2)(R^2e^{2j\beta}+2Re^{j\beta}s+5)}$$

$$= \frac{52}{(R^3e^{+j\beta})} = \frac{1}{\infty\, e^{+3j\beta}} = 0e^{-3j\beta}$$

Curve B, C, D maps to the same point B at the origin in the GH plot. D to A (-j∞ to jo) is the mirror image of the polar plot.

We need to see if (-1, 0) is enclosed by the GH map.

$$G(s) = \frac{52}{(s+2)(s^2+2s+10)} = \frac{52}{(s^3+4s^2+9s+10)}$$

$$G(j\omega) = \frac{52}{(10-4\omega^2)+j\omega(9-\omega^2)}$$

Imaginary part of G(jω) is zero where $\omega = 3$, and G(jω) at $\omega = 3$ is -2

Hence GH map cuts the real axis at (-2,0). That means (-1,0) is encircled, also N = 2 clockwise.
By inspection of G(s), we can see that there are no poles in the right half plane (or in Nyquist contour) ie.,$N_p = 0$
Thus $N_z = N + N_p$ = 2+0 = 2
It means that there are 2 roots of the characteristic equation which lie in right half plane or have roots with positive real parts which make the system unstable. The system is unstable.

Example 2: (Type 1 system)
GH = $\frac{40}{s(s+1)(s+4)}$
There is a pole at origin. To overcome this, we draw a circle of small radius ϵ, to exclude Pole at origin (s=0). The Nyquist contour is now A-B-C-D-E-F-A, where E-F-A is the semicircle of radius ϵ, drawn to exclude the pole at the origin.

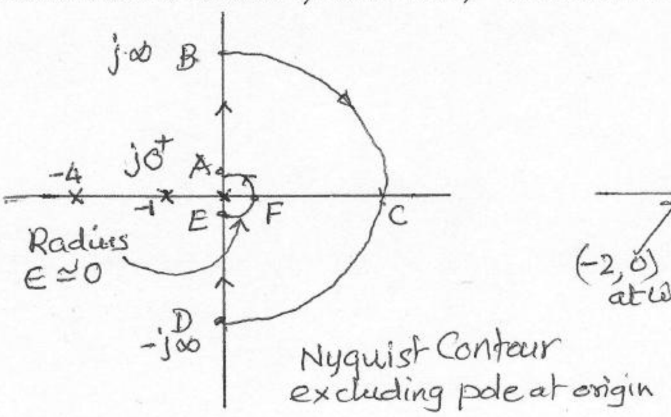
For A to B part of s-contour, in the mapping of GH, ω varies from 0^+ to ∞, which is nothing but the polar plot. It is shown as AB in GH plane. In B to C to D, R (=∞) varies from +90, 0, -90

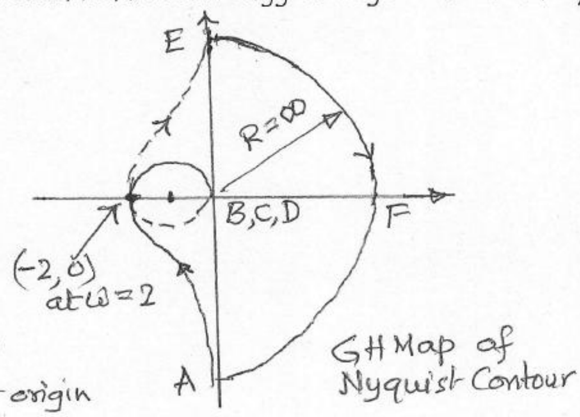
$$GH(j\omega) = \frac{40}{\infty e^{j\beta}(\infty e^{j\beta}+1)(\infty e^{j\beta}s+4)} = \frac{40}{\infty e^{3j\beta}} = 0\ e^{-3j\beta}$$

Thus the map of BCD of the Nyquist Contour is just a point at the origin.
Now, map of D to E is the variation of ω from - ∞ to 0^-. Hence it is the mirror image of the polar plot (mirror image of map of A to B).

To obtain the plot of the semicircle E-F-A, we substitute s= $\epsilon e^{j\beta}$. Thus
$$GH(j\omega) = \frac{40}{\epsilon e^{j\beta}(\epsilon e^{j\beta}+1)(\epsilon Re^{j\beta}s+4)} = \frac{10}{\epsilon e^{j\beta}} = \infty\ e^{-j\beta}$$
Hence EFA (ccω) maps to a semi-circle EFA of ∞ radius.
Note that due to $e^{-j\beta}$, the path E-F-A going from -90°, 0°, 90° in Nyquist contour maps as circle of ∞ radius, going from 90°, 0°, -90°.

The Nyquist plot is as shown below:





Now to check where GH map cuts real axis,

$$GH = \frac{40}{s(s+1)(s+4)} = \frac{40}{s(s^2+5s+4)}$$

$$GH(j\omega) = \frac{40}{-5\omega^2 + j\omega(4-\omega^2)}$$

It cuts real axis when $\omega = 2$ (Im = 0), and GH value at ω=2 is -2.

Hence (-1, 0) is encircled twice in cw sense. Hence N =2.

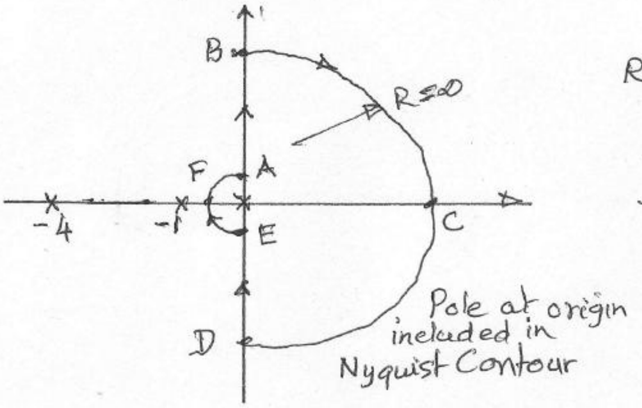
$N_p = 0$ as two roots -1, -4 are outside Nyquist contour and detour at ϵ radius has been taken to exclude s= 0 from the contour.

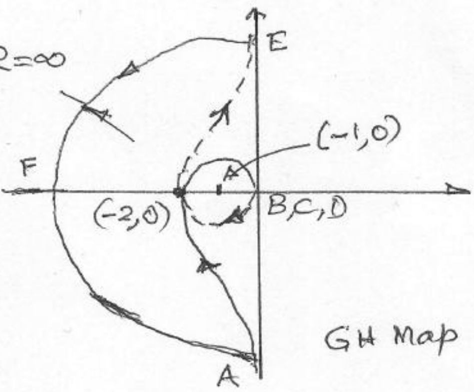
Hence $N_z = N + N_p = 2+0 = 2$

<u>Therefore the system is unstable</u>.

In the above example, we can see the result if the semi-circle of radius ϵ is drawn in cw sense to include s = 0 pole (so that N_p =1)

In that case, as $e^{-j\beta}$, the GH map leads to an ∞ semi-circle (ccw) from E to A.





Number of encirclements of (-1, 0) in GH map is 1. ie., N = 1.

$N_z = N + N_p = 1+1 = 2$

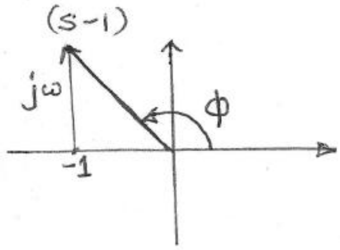
Hence the same result, giving the same number of unstable poles, is obtained.

** ** **

Examples on drawing Nyquist Map to arrive at stability.

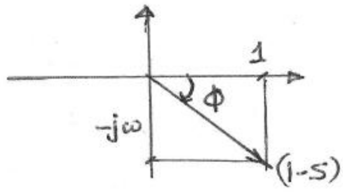
While drawing GH map, the following points may be noted:

1. For G(S)H(s) of the form K/(s+1)(s+2) or K/s(s+1), etc., the usual rules of ϕ and $|G(j\omega)|$ may be followed for ω =0 or ∞.
2. When (s+a) terms are in the numerator, the polar plot may become wavy and it will be necessary to consider a good number of ω values to draw an accurate plot.
3. When (s-1) term appears, the associated vector is in II quadrant, being (-1 + jω). The phase contribution is (180 - $\tan^{-1}\omega$), (–ve if in Denominator).



4. If (1-s) term appea4rs, or (+1-jω) the vector would be II quadrant, and phase contribution is,

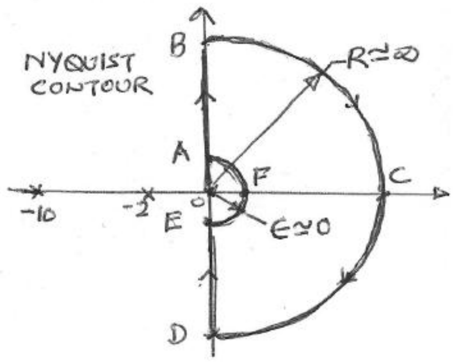
   $\Phi = -\tan^{-1}\omega$.



Example 1. If G(s) H(s) = K / s(s+2)(s+10), find range of K for stability.

(i) Nyquist Contour is drawn with a detour around origin by CCW semi-circle of radius $\varepsilon \approx 0$.

(ii) Thus all the poles of GH, ie, 0, -2, and -10 are outside the Nyquist Contour. The above detour with small radius is also drawn so that the pole 1/s or pole at the origin is outside the contour.
Hence N_p = 0.



Note: Poles of GH are the roots of the denom. of GH. Poles of GH are also poles of (1+GH)

(iii) Now we need to map each point of the Nyquist contour using the given G(s)H(s).

Nyquist's Criterion: Examples

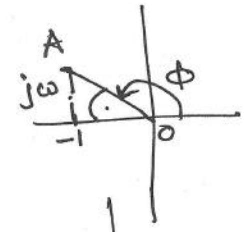
While drawing GH map, the following points may be noted:

1. When dealing with G(s) H(s) of the form $\frac{K}{(S+1)(S+2)}$ or $\frac{K}{S(S+1)}$ etc., the usual rules of ϕ and $|G(j\omega)|$ may be followed, for $\omega = 0$ and ∞.

2. When (s+a) terms are in numerator, the polar plot may become wavy and it will be necessary to consider a good no. of ω values to draw an accurate plot.

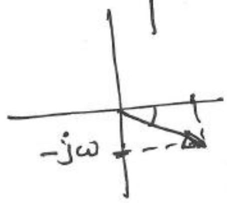
3. When (s-1) term appears in G(s)H(s), the associated vector is in IV quadrant (being $-1, j\omega$), then phase angle contribution is

$$\phi = 180^\circ - \tan^{-1} j\omega \quad \text{(if term is in Numerator)}$$



4. If there is a term $(1-s)$, or $(+1-j\omega)$ and phase contribution will be

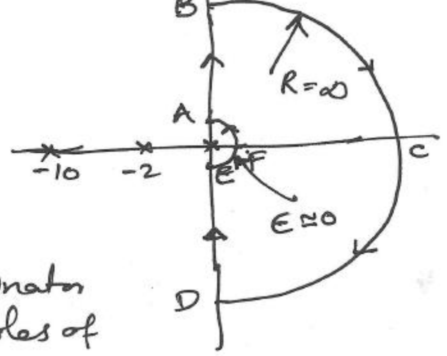
$$\phi = -\tan^{-1}\omega \quad \text{(if 1-s is on the Numerator)}$$



Examples: 1: If $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$, find range of K for stability.

(i) Nyquist's contour is drawn with a detour around origin by semi-circle of radius $\epsilon\ (\simeq 0)$ to exclude the 's' term in the denom. of G(s)H(s), from Nyquist Contour

Thus all three poles of GH i.e. 0, -2, and -10 are outside the Nyquist Contour i.e. $N_p = 0$

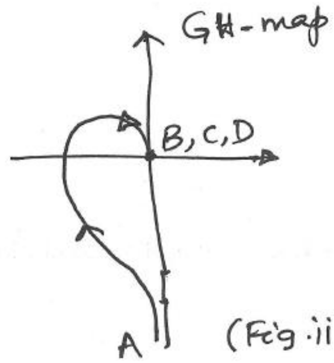


[Poles of GH are roots of denominator of GH. Poles of GH are also poles of 1+ GH]

→ Now we need to map each part of Nyquist Contour using the G(s)H(s) given.

(ii) Mapping of AB is variation of ω from 0 to ∞ in GH.

Hence it is the polar plot that starts at -90° with ∞ gain and ends in -270° (due to net 3rd order in denominator of $GH \rightarrow 3 \times -90^\circ = -270^\circ$) with 0 gain.



(Fig. ii)

(iii) BCD in Nyquist Contour is a semi circle with ∞ radius. $(R \simeq \infty)$

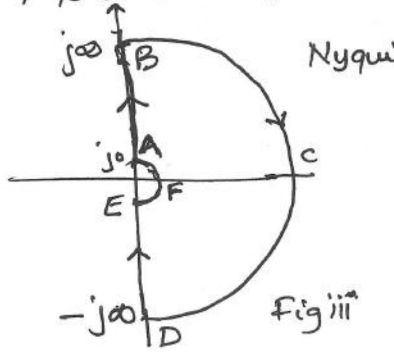
Let $s = Re^{j\theta}$ where θ varies from $+90^\circ$ to 0 to -90° clockwise

Over B, C, D

$$G(s)H(s) = \frac{K}{s(s+2)(s+10)}$$

$$= \frac{K}{Re^{j\theta}(Re^{j\theta}+2)(Re^{j\theta}+10)}$$

$$= \frac{K}{R^3 e^{3j\theta}} = 0 \cdot e^{-3j\theta}$$

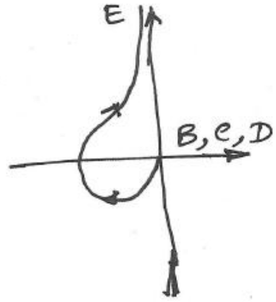


Nyquist Contour (NC)

Fig iii

Thus BCD maps as a point at the origin of GH map (Fig. ii)

(IV) DE in NC is variation of s from $-j\infty$ to $j0^-$. Hence it is the mirror image of the polar plot (map of AB)



(V) To map EFA, let $s = \epsilon e^{j\theta}$ where $\epsilon \simeq 0$.

Thus $$G(s)H(s) = \frac{K}{\epsilon e^{j\theta}(\epsilon e^{j\theta}+2)(\epsilon e^{j\theta}+10)}$$

$$= \frac{K}{\epsilon e^{j\theta} \times 2 \times 10}$$ (as $\epsilon e^{j\theta}$ is small compared to 2 and only 2 is retained in $(s+2)$ etc.)

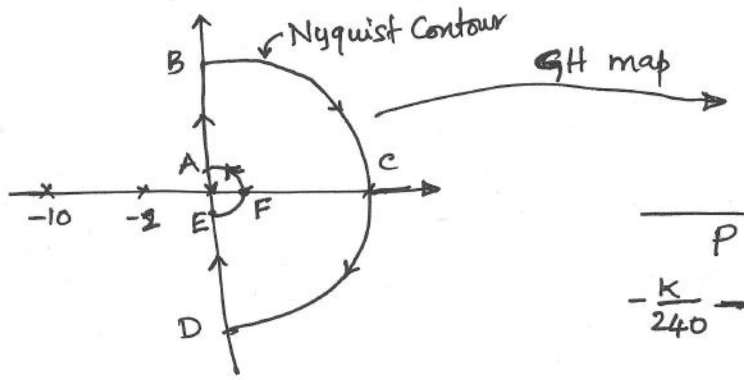
$$= \frac{K}{20\,\epsilon} e^{-j\theta} = \infty\, e^{-j\theta}$$

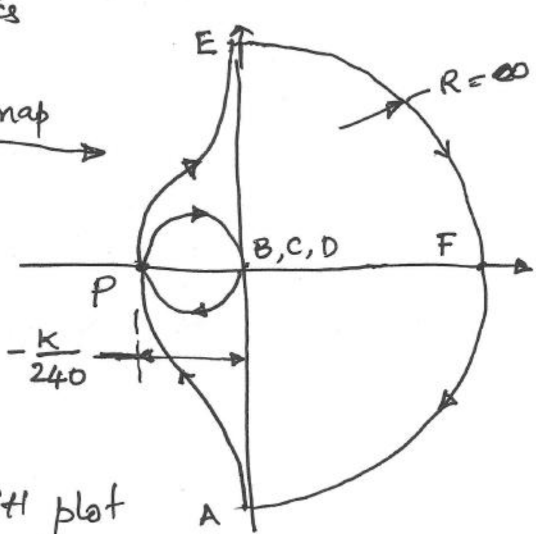
Hence EFA map of NC is a semicircle of ∞ radius. ~~from~~

Due to $-j\theta$ in mapping, as EFA in NC varies from -90° to 0 to 90° (ccw), the mapping varies from E to F to A i.e. $+90^\circ$ to 0 to -90° (cw) at ∞ radius in the GH map.

Now complete GH map can be drawn.

(vi) The complete GH map of NC is





(vii) Next step is check where the GH plot cuts the real axis. If it cuts at a point to the right side of (-1, 0), then the number of encirclements of (-1, 0) will be zero. So that $N = N_z - N_p$ or $N_z = N + N_p$, from which N_z the roots of char. equ., $1 + GH = 0$, lying in the right half plane can be evaluated. If $N_z \neq 0$, then the C. Eqn. has roots with positive real parts, and system would be unstable.

To find intersection of polar plot with real axis, we need to identify the value of ω which makes the Imaginary Part $= 0$.

Thus
$$G(s) = \frac{K}{s(s+2)(s+10)} = \frac{K}{s(s^2+12s+20)} = \frac{K}{12s^2+s(s^2+20)}$$

$$G(j\omega) = \frac{K}{-12\omega^2 + j\omega(20-\omega^2)} \quad \text{—— (1)}$$

Thus Imag. Part of GH will be zero when $20 = \omega^2$

or $\omega = \sqrt{20}$ rad/sec.

Substituting in Eqn. (1), we get $GH = \dfrac{K}{-12\omega^2} = \dfrac{K}{-12 \times 20} = -\dfrac{K}{240}$.

Thus polar plot intersects the real axis at $P\ (= -K/240)$.

Hence, if $\boxed{K > 240}$, $K/240 > 1$ so that P ~~encir~~ is to the left of (-1, 0).

∴ there are 2 clockwise encirclements, ie $N = 2$

$N_z = N + N_p = 2 + 0 = 2$.

∴ There are two unstable roots. — The system is unstable for $K > 240$.

Similarly, for $\boxed{K < 240}$, P lies on right of (-1, 0), with no encirclements.

∴ $N = 0$; $N_z = N + N_p = 0 + 0 = 0 \rightarrow$ No unstable roots $\rightarrow$ Stable system

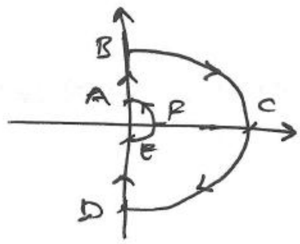
Ex. 2 : If the OLTF of a system is $G(s)H(s) = \dfrac{(1+5s)}{s^2(1+s)(1+2s)}$

Comment on the Stability of the closed-loop system using Nyquist's Criterion.

(i) Due to s^2 in denominator, take ϵ radius detour in NC.

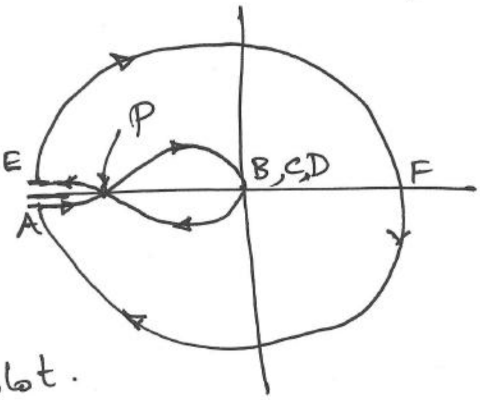
The polar plot starts at -180° and ends in -270° asymptote.

i.e. $90^\circ \times$ (1 term Nr. − 4th order in Denom.)



- BCD of NC has ∞ radius and maps as a point since

$$G(s) = \frac{(1+5Re^{j\theta})}{R^2e^{j\theta}(1+Re^{j\theta})(1+2Re^{j\theta})} = \frac{5/2}{R^3}e^{-j3\theta} = 0\,e^{-j3\theta}$$



- DE is the mirror image of AB map i.e. of polar plot.

For EFA map, let $s = \epsilon e^{j\theta}$

so that $G(s) = \dfrac{1+5\epsilon e^{j\theta}}{\epsilon^2e^{2j\theta}(1+\epsilon e^{j\theta})(1+2\epsilon e^{j\theta})} = \dfrac{1}{\epsilon^2e^{2j\theta}\times 1\times 1} = \infty e^{-2j\theta}$

Thus EFA of ϵ radius going from −90, 0, +90 maps as EFA of infinite radius from −180, 0 to +180, clockwise.

The Nyquist plot or map is completed.

- We need to identify where point P is to check encirclements of (−1, 0)

$G(s) = (1+5s)/s^2(1+3s+2s^2) = (1+5s)/s^2(1+2s^2+3s)$

$$G(j\omega) = \frac{(1+5j\omega)}{-\omega^2(1-2\omega^2+3j\omega)}$$

$$= \frac{(1+5j\omega)}{-\omega^2(1-2\omega^2+3j\omega)} \times \frac{1-2\omega^2-3j\omega}{1-2\omega^2-3j\omega}$$

Imaginary part of numerator $= 5\omega(1-2\omega^2) - 3\omega = 2\omega(1-5\omega^2)$

Hence P occurs at $1-5\omega^2 = 0$ or $\omega = \sqrt{1/5}$, where

$$-|GH| = \frac{\sqrt{1+25\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = 25/3 = \underline{8.33} \text{ at P}$$

- Hence (−1, 0) is ~~inside~~ to the right of P. (between P and origin).
- There are two clockwise encirclements. $N_z = N + N_p = 2 + 0 = \underline{2}$

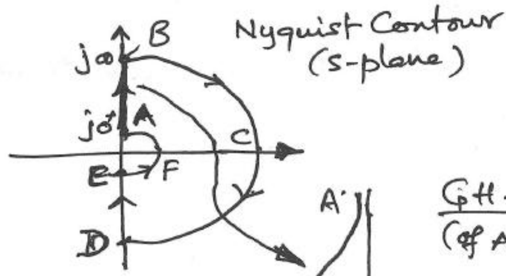
Hence system is unstable with two unstable roots of CE.

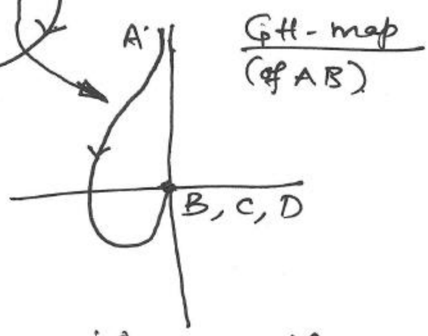
Example 3: For OLTF, $G(s) = \frac{K(1+s)^2}{s^3}$, find range of K for stability of the CL system.

$$G(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K(1-\omega^2+j2\omega)}{-j\omega^3} = \frac{-K(1-\omega^2)}{j\omega^3} - \frac{2K}{\omega^2} \quad \text{---} (1)$$

Also $|G(j\omega)| = \frac{K\sqrt{1+\omega^2}\sqrt{1+\omega^2}}{\omega^3}$

and $\phi = 2\tan^{-1}\omega - 270°$





(i) Map of AB is the polar plot in GH map
- Plot starts at $-270°$ at $\omega \simeq 0$ where $|GH| \simeq \infty$. It ends at ($\omega \simeq \infty$) $\phi = 2\times 90 - 270 = -90°$ where $|GH| = 0$.

(ii) For map of BCD, let $s = Re^{j\theta}$, $R \simeq \infty$,

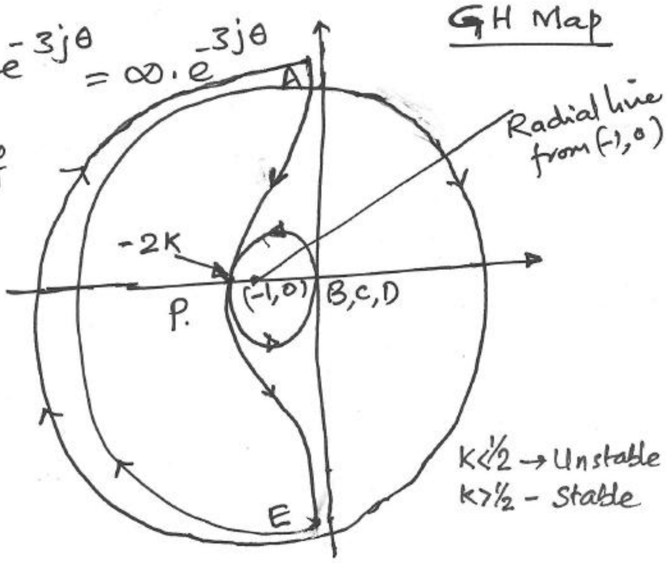
then $G(s) = \frac{K(1+Re^{j\theta})^2}{(Re^{j\theta})^3} = \frac{K.R^2e^{2j\theta}}{R^3e^{3j\theta}} = \frac{K}{R}e^{-j\theta} = 0 \cdot e^{-j\theta}$

Thus BCD of s-plane maps as a point at the origin of GH map.

(iii) DE map is the mirror image of AB map.

(iv) For EFA map, let $s = \epsilon e^{j\theta}$ so that

$$G(s) = \frac{K(1+\epsilon e^{j\theta})^2}{(\epsilon e^{j\theta})^3} = \frac{K}{\epsilon^3}e^{-3j\theta} = \infty \cdot e^{-3j\theta}$$



Thus EFA of NC maps as a circle of ∞ radius. Also due to $e^{-3j\theta}$, $-90, 0, +90$ of EFA maps over $+270, 0, -270°$ as shown.

- Note that $180°$ of ccw rotation of EFA maps to $3\times180° = 540°$ in cw.

- To locate ω where polar plot cuts the real axis, from Equ.(1), $\omega = 1$, where $|G(j\omega)| = \frac{K\sqrt{2}\sqrt{2}}{1} = 2K$. (or at $-\frac{2K}{\omega^2} = -2K$ in Equ.(1))

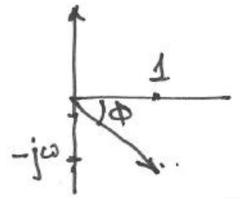
K < ½, point (-1,0) is to left of point P. and N = 2. (Unstable)

K > ½, point (-1,0) is to the right of point P. N = 0 in this case (To count encirclements draw a radial line from (-1,0). It is cut by the GH map once in cw drn. and once in ccw drn. Hence N = 1-1 = 0)

$\therefore N_z = N + N_p = 0 + 0 = 0$. Hence System is stable for K > ½.

Example 4: Comment on the Stability of CL System which has OLTF given by $G(s) = \dfrac{5}{s(1-s)}$

$$|G(j\omega)| = \left|\frac{5}{j\omega(1-j\omega)}\right| = \frac{5}{\omega\sqrt{\omega^2+1}}$$



& $\phi = -90 - (-\tan^{-1}\omega) = -90^\circ + \tan^{-1}\omega$

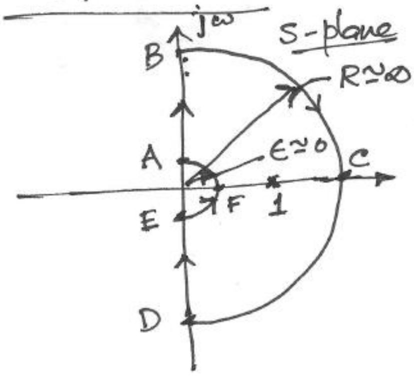
$\phi = -\tan^{-1}\omega$

Hence at $\omega \cong 0$, $|G(j\omega)| = \infty$ & $\phi = -90^\circ$

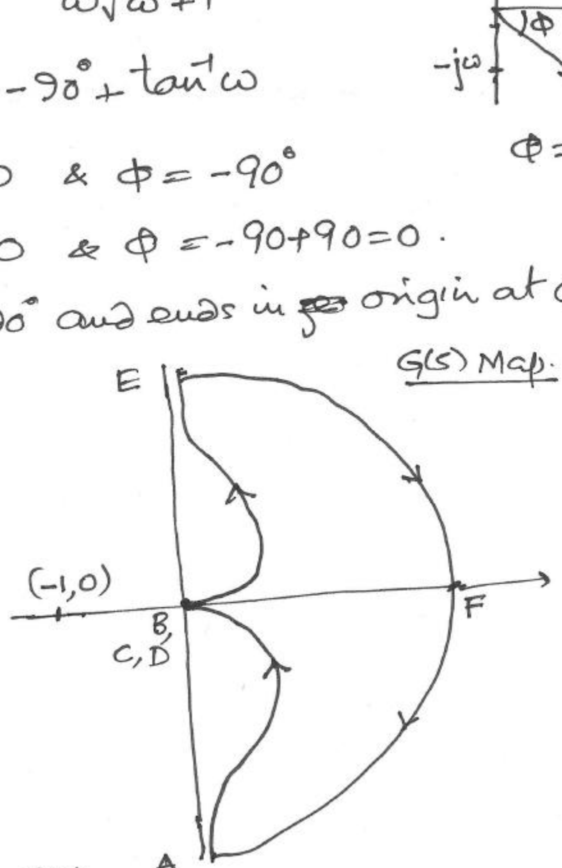
at $\omega \simeq \infty$, $|G(j\omega)| = 0$ & $\phi = -90+90 = 0$.

Hence polar plots starts at -90° and ends in origin at 0° angle.

Complete Nyquist Plot



G(s) Map.



- AB maps in IV quadrant in G(s) plane as shown.
- For BCD, letting $s = Re^{j\theta}$ ($R \to \infty$),

$$G(j\omega) = \frac{5}{Re^{j\theta}(1-Re^{j\theta})} = \frac{5}{-R^2 e^{2j\theta}} = 0\, e^{-2j\theta} \rightarrow \text{maps as point at origin.}$$

- EFA, let $s = \epsilon e^{j\theta}$, ($\epsilon \simeq 0$) gives

$$G(j\omega) = \frac{5}{\epsilon e^{j\theta}(1-\epsilon e^{j\theta})} = \frac{5}{\epsilon e^{j\theta}} = \infty . e^{-j\theta}$$

$\therefore$ EFA of s-plane maps as circle, in CW direction, of ∞ radius.

The complete Nyquist map is as shown in the G(s)-map.

It is seen that $(-1,0)$ is not encircled by G(s) map. i.e $N = 0$

- Nyquist Contour has a pole $s=1$ of $G(s)$ enclosed. i.e $N_p = 1$.

$\therefore \quad N_z = N + N_p = 0 + 1 = 1$

which means one root of CE lies in Nyquist Contour (or RHP).

$\therefore$ System is Unstable

Example 5 : $G(s) = \dfrac{s+2}{(s+1)(s-1)}$

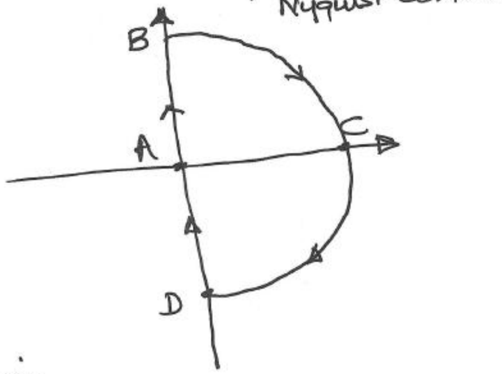
We have

$$\phi = \tan^{-1}\frac{\omega}{2} - \tan^{-1}\omega - (180 - \tan^{-1}\omega) = \underline{-180^\circ + \tan^{-1}\omega/_2} \quad — (1)$$

(Since $s-1$ is in II quadrant).

$$|G(s)| = \frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+1}} = \frac{\sqrt{\omega^2+4}}{\omega^2+1} \quad —— (2)$$

s-plane Nyquist Contour



- Map of AB in GH plane is the polar plot

At A: as $\omega \simeq 0$, $\phi = -180$ (from Eqn. 1)

$|GH| = 2$ (from Eqn. 2)

At B: as $\omega \simeq \infty$; $\phi = -180+90 = -90^\circ$

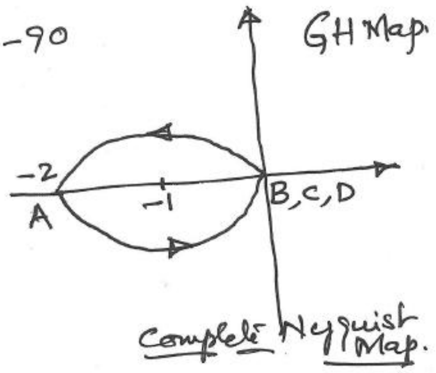
$|GH| = \frac{1}{\omega} = 0.$

A starts at $(-2,0)$ and ends at origin at -90°.

Some intermediate values also may be calculated.

ω	0	0.4	1	2	10	∞
ϕ	2	1.76	1.12	0.57	0.1	0
$\lvert GH \rvert$	-180	-168	-153	-135	-101	-90

GH Map



Complete Nyquist Map.

For map of BCD, let $s = Re^{j\theta}$, $R \simeq \infty$,

$$GH(s) = \frac{(Re^{j\theta}+2)}{(Re^{j\theta}+1)(Re^{j\theta}-1)}$$

$$= \frac{1}{R}e^{-j\theta} = 0\,e^{-j\theta}.$$

- Hence BCD maps as a point at origin of GH plane.
- DA being from $-j\infty$ to 0 in s plane - leads to a mirror image of AB map - and is as shown.

N = No. of encirclements of $(-1,0)$ in GH map $= -1$, being CCW.

$\therefore N_z = N + N_p = -1 + 1 = 0.$

i.e. No roots of the CE $(1+GH=0)$ in Nyquist Contour i.e. Right half plane.

$\therefore$ System is Stable.

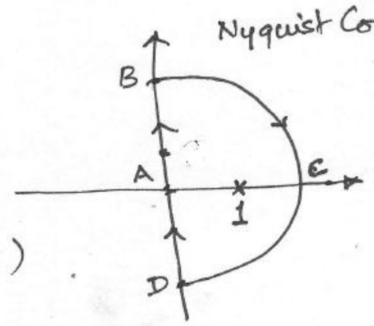
Example 6: $GH(s) = \dfrac{K(1+0.5s)(1+s)}{(1+10s)(s-1)}$; find range of K for stability.

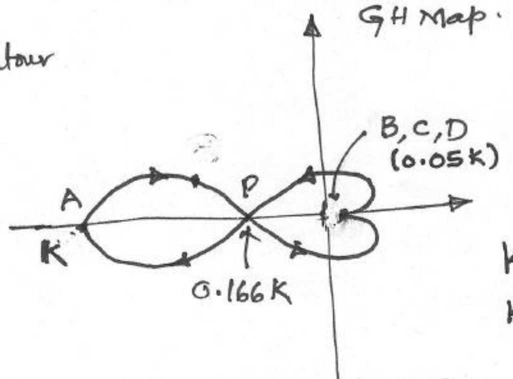
$$|G(j\omega)| = \frac{K\sqrt{1+(0.5\omega)^2}\sqrt{1+\omega^2}}{\sqrt{1+100\omega^2}\sqrt{1+\omega^2}} = \frac{K\sqrt{1+0.25\omega^2}}{\sqrt{1+100\omega^2}} \quad \text{— (1)}$$

$$\phi = \tan^{-1}0.5\omega + \tan^{-1}\omega - \tan^{-1}10\omega - (180 - \tan^{-1}\omega) \quad \text{— (2)}$$

$$= \tan^{-1}0.5\omega + 2\tan^{-1}\omega - \tan^{-1}10\omega - 180$$

(Np=1 due to s-1)





K>6 → Stable

K<6 → Unstable.

AB Map : It is the polar plot and from Eqns. (1) & (2)

ω	0	0.1	0.5	1	2	10	100	∞
$\lvert G(j\omega)\rvert$	K	0.71K	0.2K	0.11K	0.07K	0.051K	0.050	0.05K
ϕ	-180	-210	-191	-147	-95	-22	-2.2	0

BCD: $G(s) = \dfrac{K(1+0.5Re^{j\theta})(1+Re^{j\theta})}{(1+10Re^{j\theta})(Re^{j\theta}-1)} = 0.05K.$

DA : - It is the mirror image of AB Map

Crossing of Real axis is when Imag. part = 0.

$$G(s) = \frac{K(1+0.5s)(1+s)}{(1+10s)\cdot(s-1)} = \frac{K(1+1.5s+0.5s^2)}{10s^2-9s+1}$$

$$\frac{G(j\omega)}{K} = \frac{(1-0.5\omega^2)+j\times1.5\omega}{-(1+10\omega^2)-9j\omega} \times \frac{-(1+10\omega^2)+j9\omega}{-(1+10\omega^2)+j9\omega}$$

Imag. part $= 9(1-0.5\omega^2) - 1.5\omega(1+10\omega^2)$

$= \omega(9-4.5\omega^2-1.5-15\omega^2) = \omega(7.5-19.5\omega^2)$

Imag. part $=0 \longrightarrow \omega = \sqrt{7.5/19.5} = 0.62$

At $\omega = 0.62$, $|GH(j\omega)| = 0.166K$ (from Eqn. (1))

→ At P where $|GH| = 0.166K$, if $K>6$, then $(-1,0)$ is to the right of P. Then $N=-1$; → $N_z = N+N_p = -1+1 = 0$ System is Stable. If $K<6$, P is to left of $(-1,0)$ Then $N=1$; $N_z = N+N_p = 1+1 = 2$. System is unstable