Course Material

Engineering Mechanics

Topic: Friction

by

Dr.M.Madhavi,

Professor,

Department of Mechanical Engineering,

M.V.S.R.Engineering College,

Hyderabad.

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PART-I

I. INTRODUCTION

1.0 Definition:

Friction is defined as the contact resistance exerted by one body upon a second body when the second body moves or tends to move past the first body.

Friction is a retarding force always acting opposite to the motion or tendency to move.

Whenever a tendency exists for one contacting surface to slide along another surface the friction forces developed are always in a direction to oppose this tendency.

a) In some types of machines we want to minimize the retarding effect of friction forces.
   Examples : bearings of all types, power screws, gears, flow of fluid in pipes, propulsion of aircraft, missiles through the atmosphere.

b) In some situations we wish to maximize the effect of friction as in brakes, clutches, belt drives and wedges.
   Wheeled vehicles depend on friction for both starting and stopping and ordinary walking depends on friction between the shoe and ground.

2.0 Types of Friction:

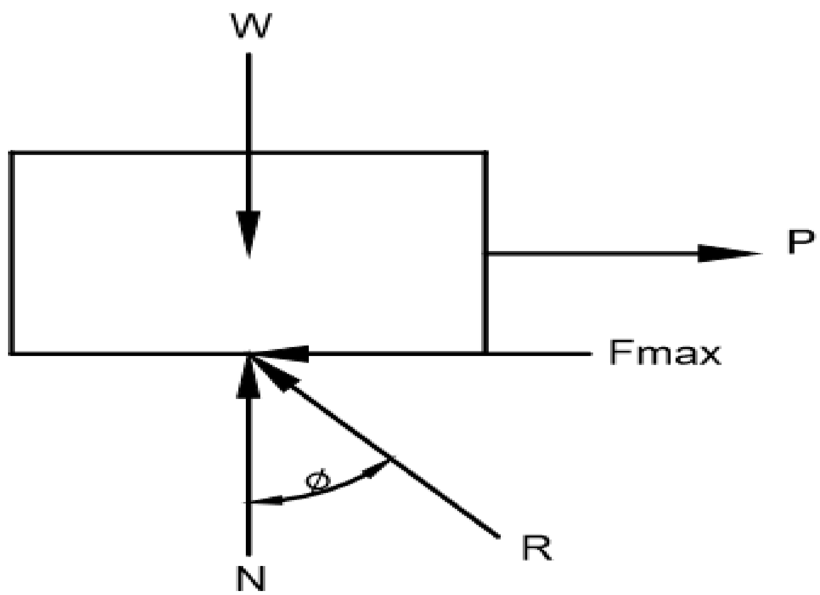
Dry Friction: Dry friction develops when the unlubricated surface of two solids are in contact under a condition of sliding or a tendency to slide. This type of friction is also known as Coulomb friction.

Fluid Friction: Fluid friction is developed when adjacent layers in a fluid (liquid or gas) are moving at different velocities.

3.0 Mechanism of Friction:

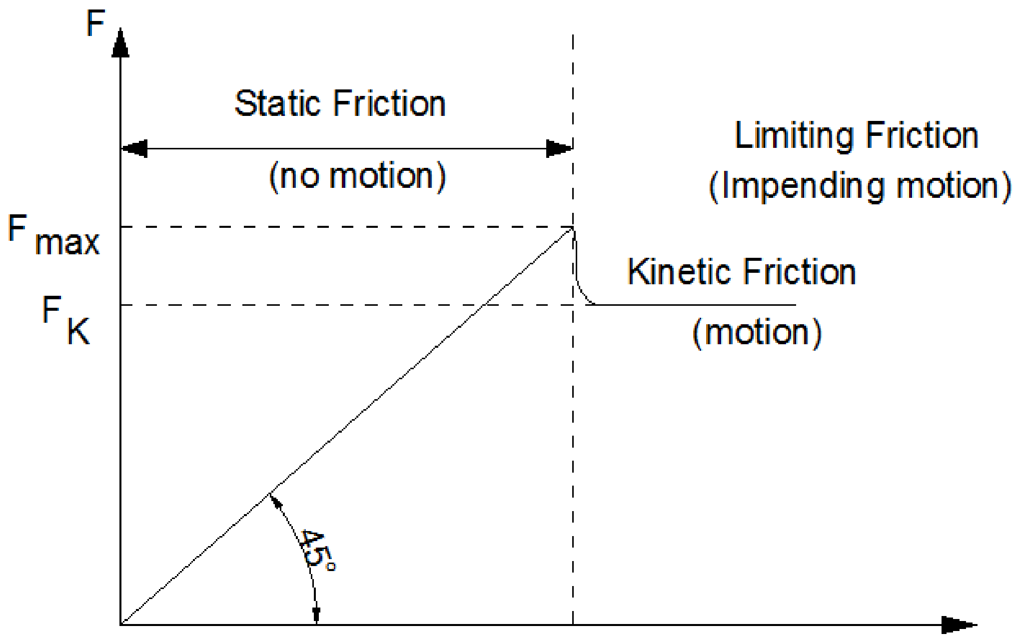
Friction exists primarily because of the roughness of the contact surface. Consider a block of weight w resting on a horizontal surface. The contacting surface possesses a certain amount of roughness. Let P be the horizontal force applied which will vary continuously from zero to a value sufficient to just move the block and then to maintain the motion. The free body diagram of the block shows active forces (i .e, applied force P and weight of block w) and reactive forces (i. e, normal reaction N and tangential frictional force F).

Frictional force F has the remarkable property of adjusting itself in magnitude equal to the applied force P till the limiting equilibrium condition.



The above discussion can be represented by a graph with applied force P v/s frictional force F as shown in Fig.

Referring the graph we may now recognize three distinct types of problems. Here, we have static friction, limiting friction and kinetic friction.



P(Applied Force)

1. **<u>Static Friction:</u>**

   If in the problem there is neither the condition of impending motion nor that of motion then to determine the actual force, we first assume static equilibrium and take F as frictional force required to maintain the equilibrium condition.

   Here, we have three possibilities.

   (i) $F < F_{max}$ = Body is in the static equilibrium condition which means body is purely at rest.

   (ii) $F = F_{max}$ = Body is in limiting equilibrium condition which means impending motion and hence $F = F_{max} = \mu_s N$ is valid equation.

   (iii) $F > F_{max}$= Body is in motion which means $F = F_k = \mu_k N$ is valid equation the condition is impossible, since the surfaces cannot support more force than the maximum frictional force. Therefore, the assumption of equilibrium is invalid, the motion occurs.

2. **Limiting Friction:** The condition of impending motion is known to exist. Here a body which is in equilibrium is on a verge of slipping which means the body is in limiting equilibrium condition. "It is the maximum value of friction force that the surface can exert on the block and is designated as $F_{max.}$".This mainly depends on roughness of the materials of the surfaces and of the normal contact force which these surfaces exert on each other.

   $F_{max} = \mu_s N$ is valid equation.

3. **Kinetic Friction:** The condition of relative motion is known to exist between the contacting surfaces. So, the body is in motion.

   Kinetic friction takes place $F_k = \mu_k N$ is valid equation.

4.0 LAWS OF FRICTION

1. The frictional force is always tangential to the contact surface and acts in the direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have the impending motion.
3. Limiting frictional force F_{max} is directly proportional to normal reaction ($F_{max} = \mu_s N$).
4. For a body in motion, kinetic frictional force F_k developed is less than that of limiting frictional force F_{max} and the relation $F_k = \mu_k N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of speed of the body.

8. Coefficient of static friction μ_s is always greater than coefficient of kinetic friction μ_k. (μ_k may be 25% smaller than μ_s in general).

Note: μ_s & μ_k are dimensionless.

5.0 . Values of Coefficient of Friction

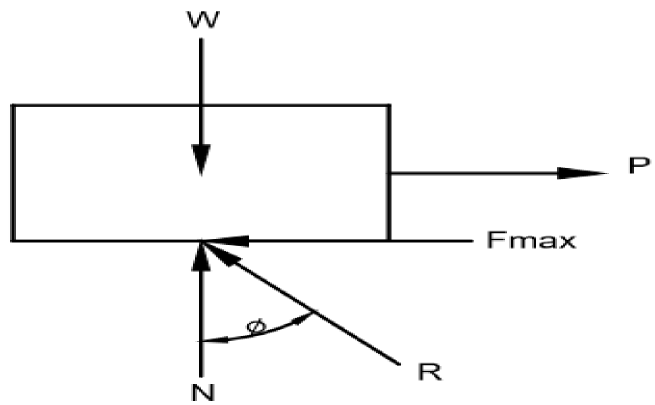
Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

6.0 Friction Angles

a) Angle of Friction:

It is the angle made by the resultant of the limiting frictional force F_{max} and the normal reaction N with the normal reactions.



Consider the block with weight W and the applied force P.

When the block is at a verge of motion, limiting frictional force F_{max}will act opposite direction of applied force and the normal reaction N will be perpendicular to the surface as shown in Fig. We can replace the F_{max} and N by resultant reaction R which acts at an angle ø to the normal reaction.The angle ø is called ***angle of friction***.

From Fig we have

$$F_{max} = R \sin ø$$

$$\mu_s N = R \sin ø \quad \text{---------} \text{ (I)} \quad (F_{max} = \mu_s N)$$

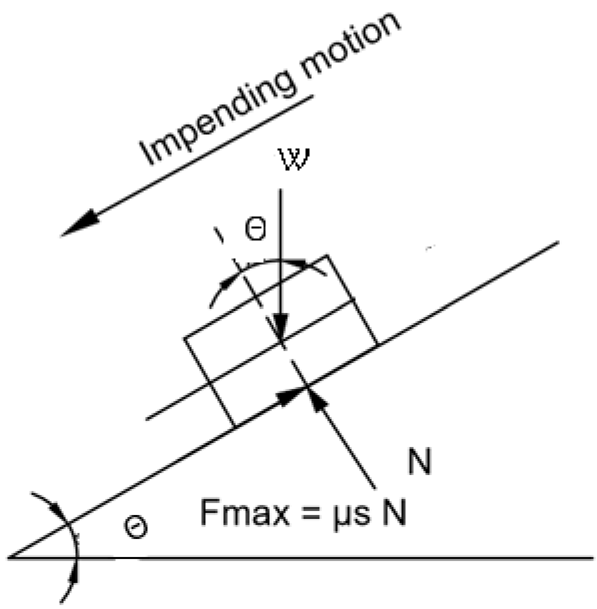
$$N = R \cos ø \quad \text{---------} \text{ (II)}$$

Dividing Eq. (I) by Eq. (II), we get

$$\tan ø = \mu_s \text{ or } ø = \tan^{-1} (\mu_s)$$

c) Angle of Repose

It is the minimum angle of inclination of a plane with the horizontal at which the body kept will just slide down on it without the application of any external force (Due to self-weight).



Consider the block with weight W is resting on an inclined plane, which makes an angleΘ with the horizontal as shown in figure. When Θ is small the block will rest on the plane. If Θ is increased gradually a slope is reached at which the block is about to start sliding. This angle Θ is called ***angle of repose***.

$$\sum F_x = 0$$

$$\mu_s N - W \sin\Theta = 0$$

$$W\sin\Theta = \mu N \qquad \text{------------ (I)}$$

$$\sum F_y = 0$$

$$N - W\cos\Theta = 0$$

$$W\cos\Theta = N \qquad \text{------------ (II)}$$

Dividing Eq. (I) by Eq. (II), we get

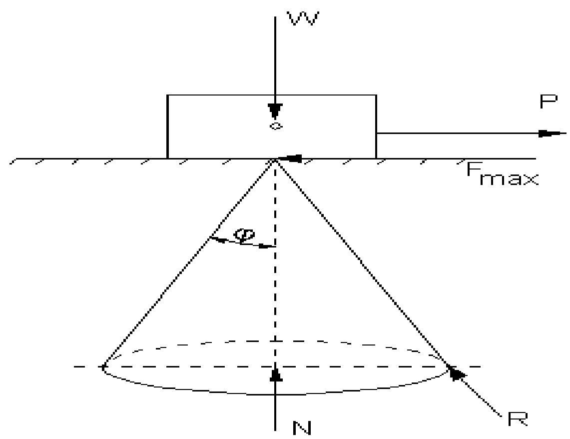
Tan Θ = μ_s

In previous discussion, we had Tanø = μ_s which shows

Angle of friction ø = Angle of Repose Θ

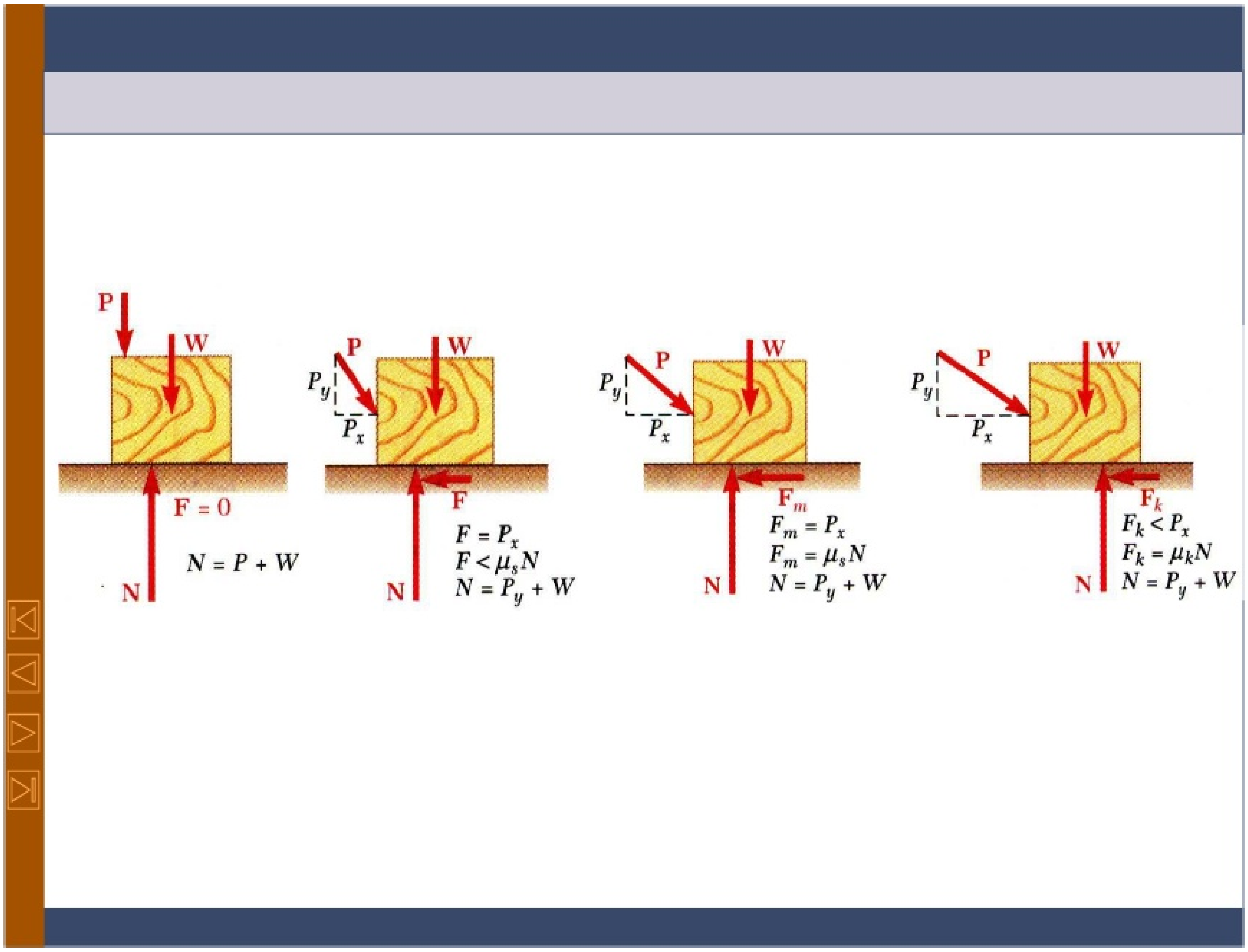
The above relation also shows that the angle of repose is independent of weight of the body it depends on μ.

Cone of Friction:



When the applied force P is just sufficient to produce the impending motion of given body, angle of friction ø is obtained which is the angle made by resultant of limiting frictional force with normal reaction as shown in Fig. If the direction of applied force P is gradually changed through 360^0, the resultant R generates a right circular cone with semi vertex angle equal to ø. This is called ***Cone of Friction.***

FOUR DIFFERENT SITUATIONS MAY OCCUR WHEN A RIGID BODY IS IN CONTACT WITH A HORIZONTAL SURFACE.



First case: No friction, Px = 0 ; F=0

Second case : No motion, Px < Fm

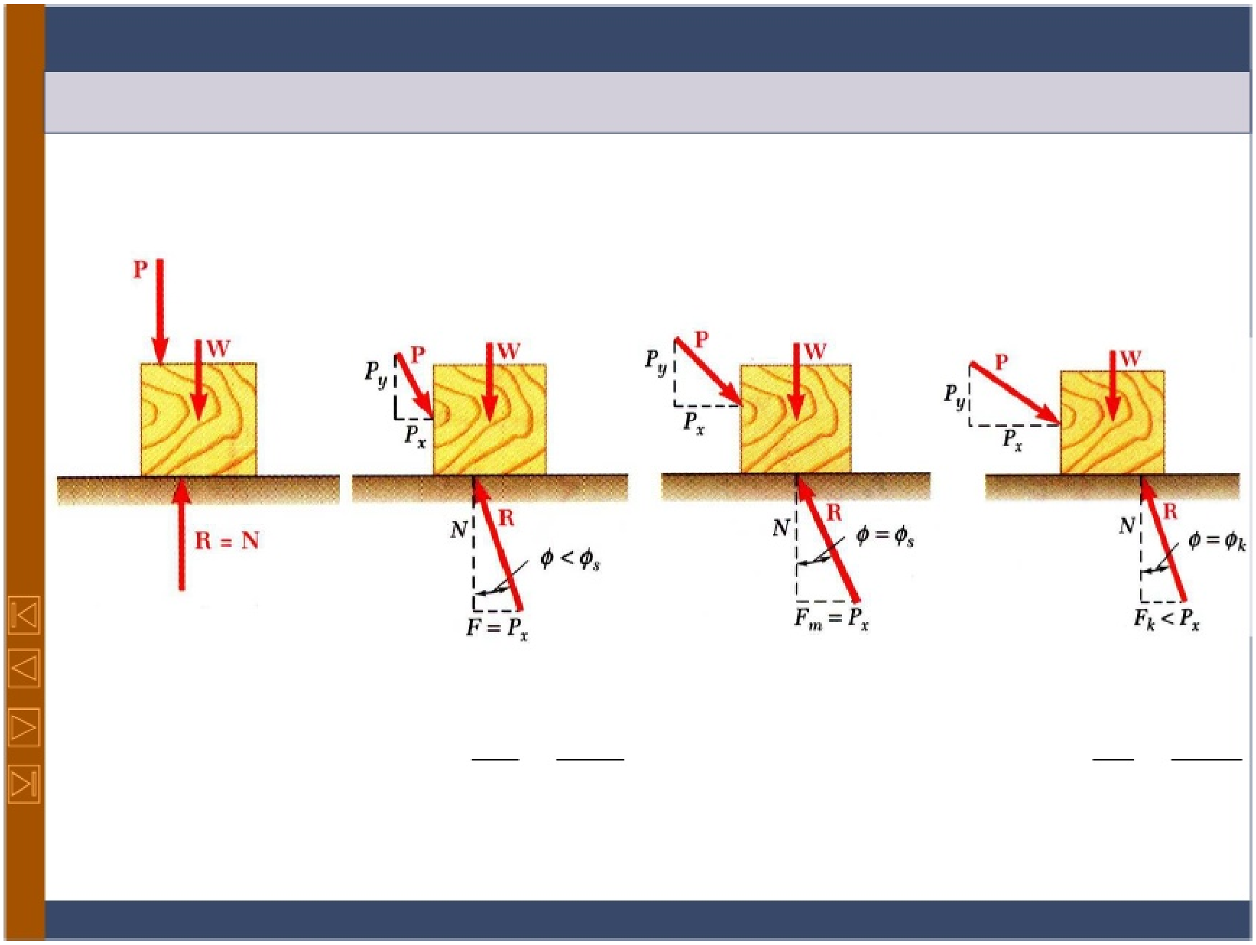
There is no evidence that the maximum frictional force has been reached.

$$F \neq Fm = \mu_s N$$

Third case: Motion impending, Px = Fm ;if the body just about to slide

F= Fm= μ_sN may be used.

Fourth case: Motion, Px > Fm ; Fm = μ_KN

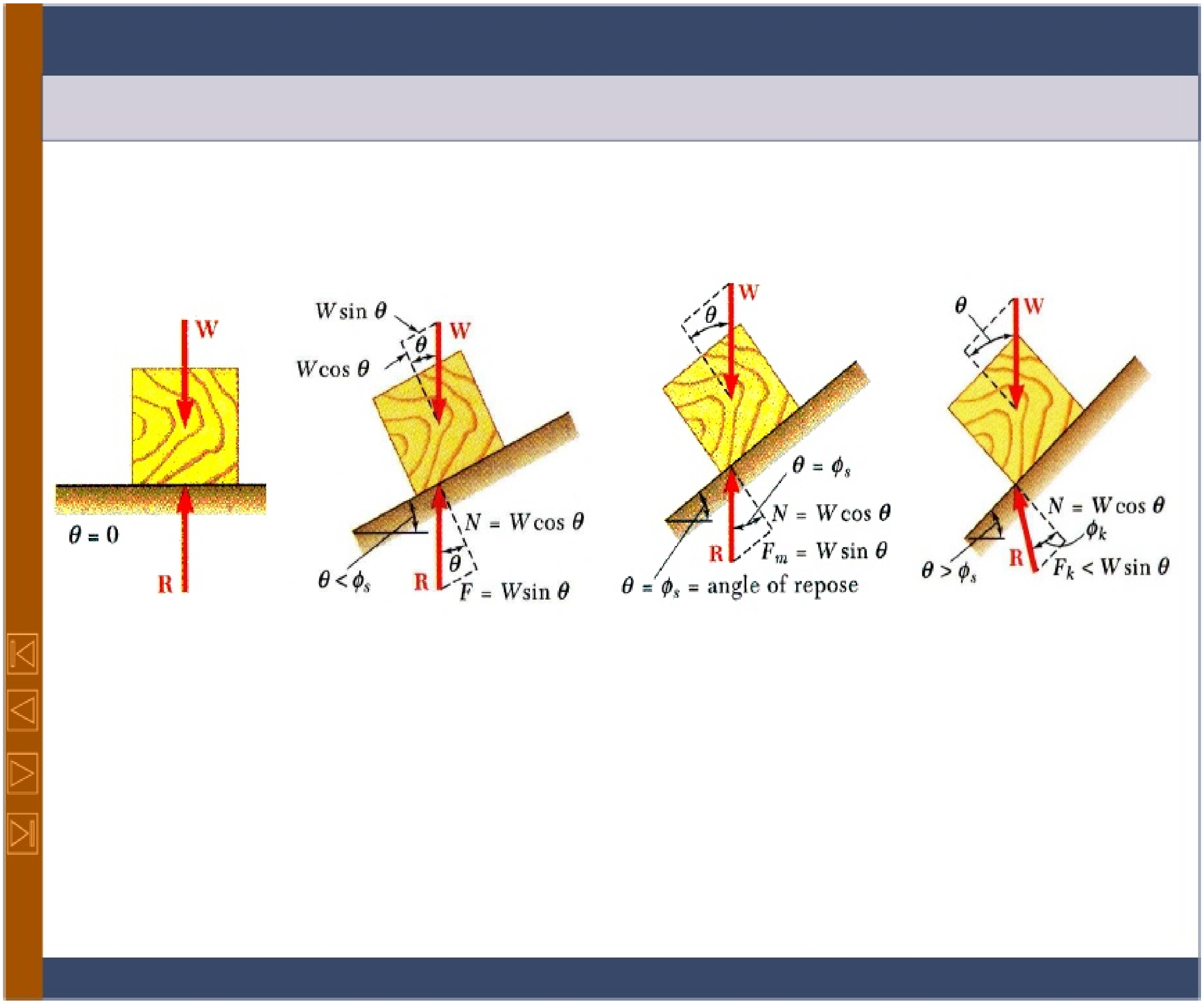


First case: **No friction**

Second case: **No motion ;** **$\tan\Phi s = Fm/N = \mu s\ N / N = \mu s$**

Third case: **Motion impending**

Fourth case: **$\tan\Phi k = Fk/N = \mu k\ N / N = \mu k$**



First case: *No friction* ; $\theta = 0$

Second case: $\theta < \Phi_s$

N=Wcos ; F=Wsinθ

$\tan\Phi = F/N = \tan\theta < \tan \Phi_s$;

$\Phi < \Phi_s$; *No motion*

Third case: $\theta = \Phi_s$

N=Wcos ; F=Wsinθ

TanΦ = F/N ; here F = Fm =μ_sN

Tan = Fm /N = μ_s = tanθ

Motion impending

Fourth case: $\theta > \Phi_s$

N=Wcosθ

Fmax = Fm =μ_sN

N tan > N tanΦ

Wsin > Fmax

$F = F_k = \mu_k N$

$\Phi = \Phi_k < \Phi_s < \theta$

R is not vertical; forces acting on block are unbalanced.

7.0 Types of Friction Problems

The first step in solving a friction problem is to identify its type

1) In the first type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping, and the friction force equals the limiting static friction . The equations of equilibrium will holds good.
2) In the second type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force F necessary for equilibrium. Three outcomes are possible:

   a) F < ()

   Here the friction force necessary for equilibrium can be supported and therefore the body is in static equilibrium as assumed.

   We emphasize that the actual friction force F is less than the limiting value (i.e,) and that F is determined by the equations of equilibrium.

   b) F = ()

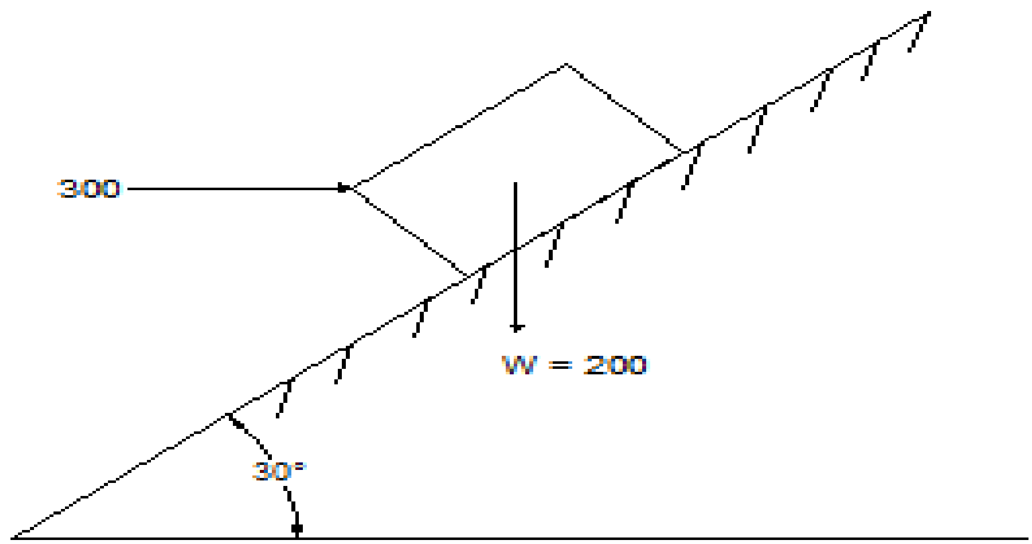
   Since the friction force F is at its maximum value , motion impends. The assumption of static equilibrium is valid.

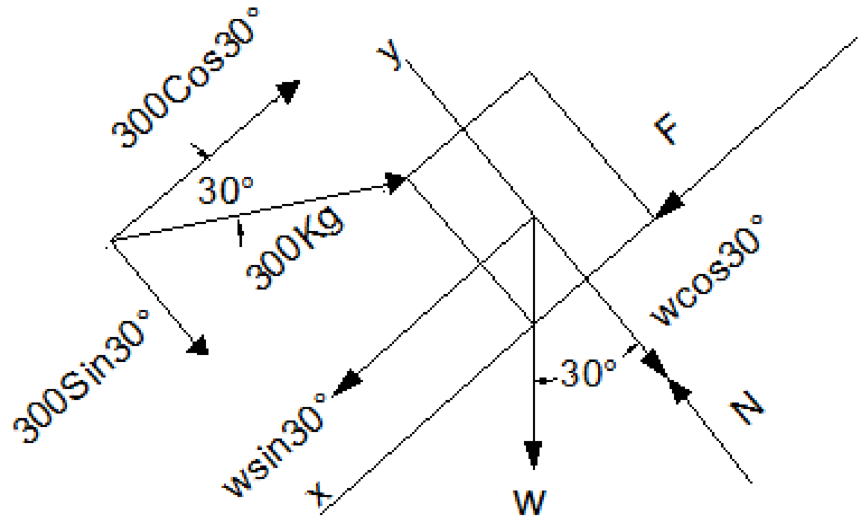
   c) F > ()

   This condition is impossible, because the surfaces cannot support more force than the maximum . The assumption of equilibrium is therefore invalid, and motion occurs. The friction force F is equal to .
3) In the third type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies.

8.0 Problems

1) Will the 200N block be held in equilibrium by the horizontal force of 300N, if μ=0.3.





Sol: Assume that 300N force is sufficient to hold the block from sliding down the plan. Let F acts down the plane.

F + 200 sin= 300 cos

F = 300 cos-200sin=160N

For balance to exist, a frictional resistance of F=160kg would be required acting down plane.

-N + 300sin + 200cos = 0

N=323N

However, the maximum value obtained (limiting friction)

$F^{i} = \mu N = 0.3 \times 323 = 97N$

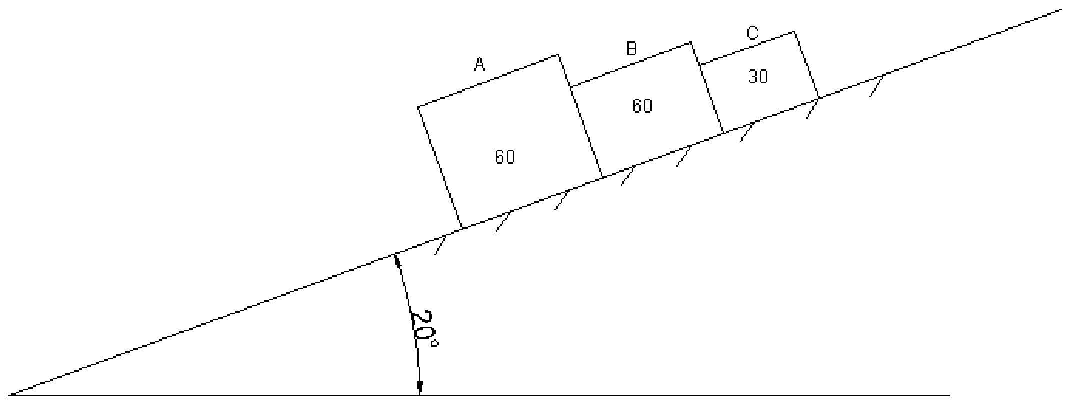
The value of F necessary to hold the block from moving up the plane is 160N.

Therefore it means that the block will move up the plane.

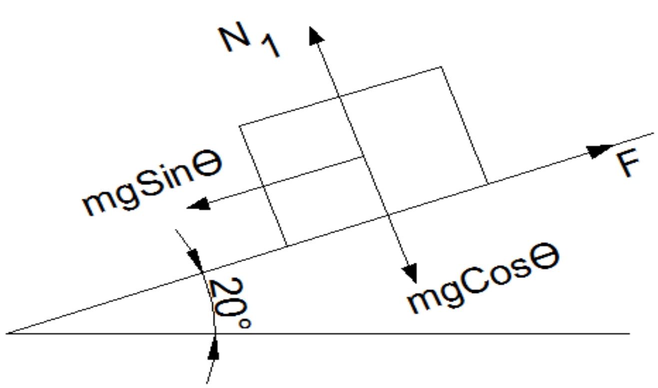
Practice Problems

1. Determine which if any of blocks will move and frictional force acting under each, for A & C
   , ; for B , . ;

   Let



Solution to Problem 1:



Consider <u>FBD of C</u>

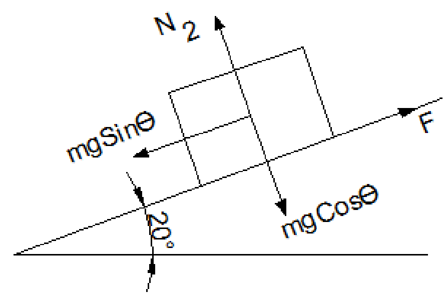
But frictional force to resist the motion in downward direction

The available frictional forces,

f > downward force

Therefore 'C' will not move.

<u>FBD of B</u>



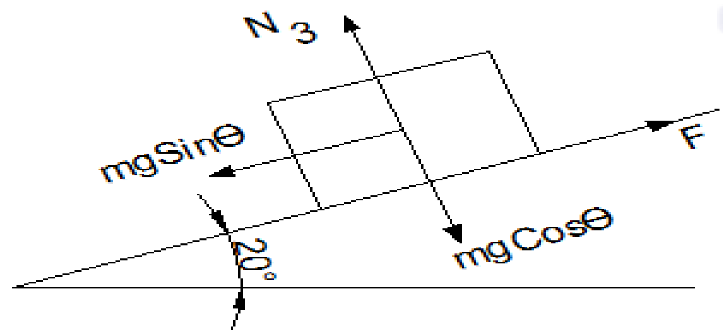
Downward force =

f < downward force

But due to greater resistance in frictional force at 'A', motion resists.

'B' will not move.

FBD of A

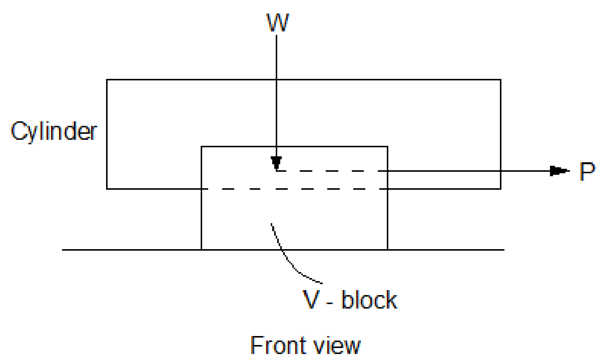


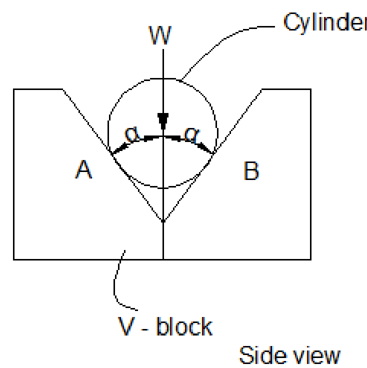
Downward force =

f > downward force

'A' will not move.

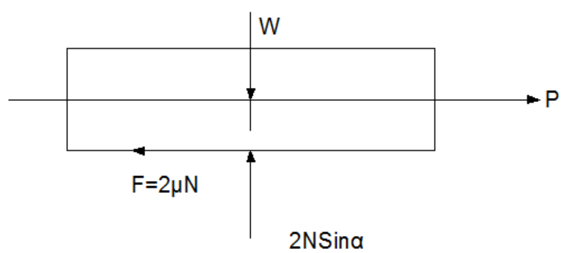
2. A straight circular cylinder of weight W rests on a V-block having an angle 2α as shown in figure. If the coefficient of friction is μ between contacting surfaces, find the horizontal force P necessary to cause slipping to impend.

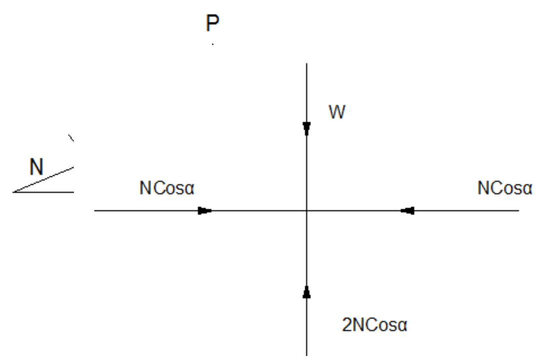




Solution to Problem2 :

FBD:





PART-II: *APPLICATIONS OF FRICTION*

1. WEDGES

2. CONNECTED BODIES

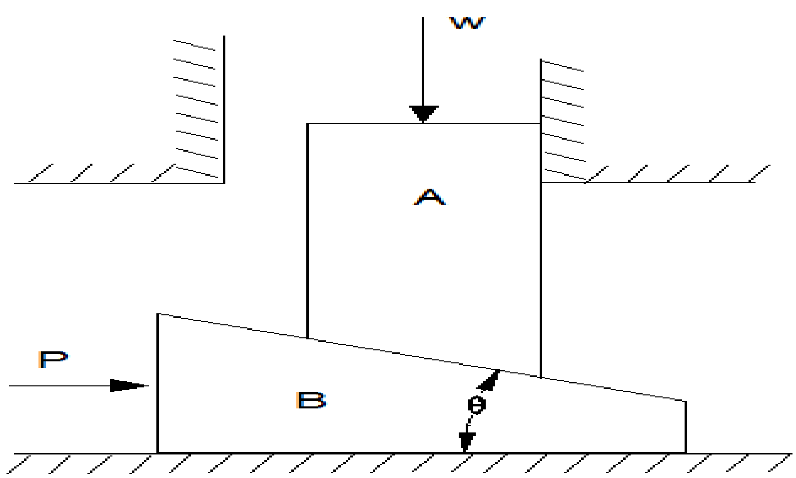
3. LADDER

4. BELTS

1. Wedges

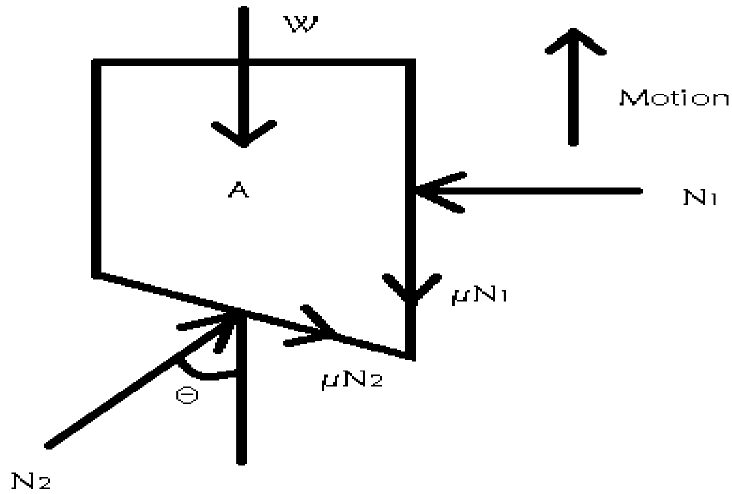
A wedge is one of the simplest and most useful machines. A wedge is used to produce small adjustments in the position of a body or to apply large forces. Wedges largely depend on friction to function.

- In the figure, the block A supports a load W and is to be raised by forcing the wedge under it.
  The contact reactions between the blocks at this common surface are not only equal and oppositely directed on the free body diagram of each block; they also act so that their tangential or frictional components along the common contact surface oppose the impending motion of each block relative to the other.
- In order for the wedge to slide out of its space, slippage must occur at both surfaces simultaneously, otherwise the wedge is self-locking.

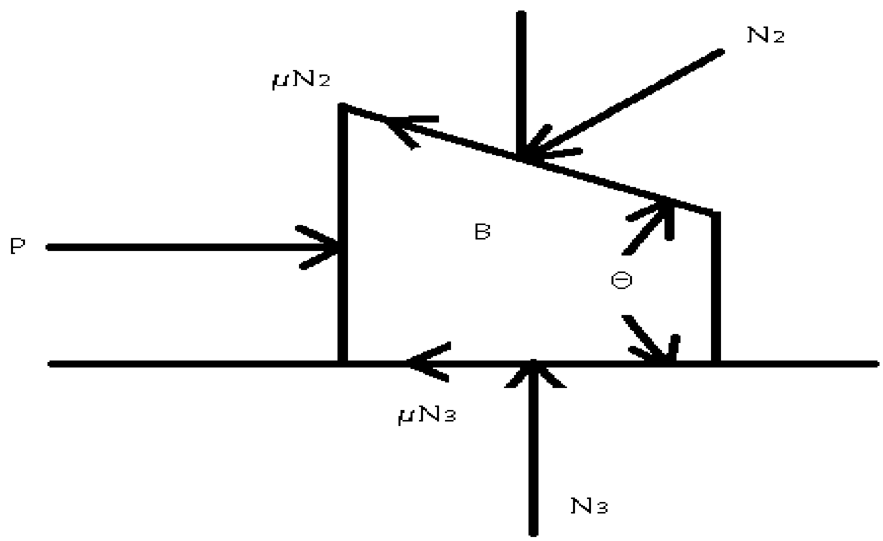


<u>Method I</u>:

<u>Equations of equilibrium using resolution method:</u>



Forces on wedge:



Wedge is considered as weightless component.

$\sum F_x = 0$

$N_1 = N_2 \mathrm{Sin}\Theta + \mu N_2 \mathrm{Cos}\Theta$

$\sum F_y = 0$

$W + \mu N_1 + \mu N_2 \mathrm{Sin}\Theta = N_2 \mathrm{Cos}\Theta$

Equilibrium equations for Wedge:

$\sum F_x = 0$

$P = \mu N_2 Cos\Theta + N_2 Sin\Theta + \mu N_3$

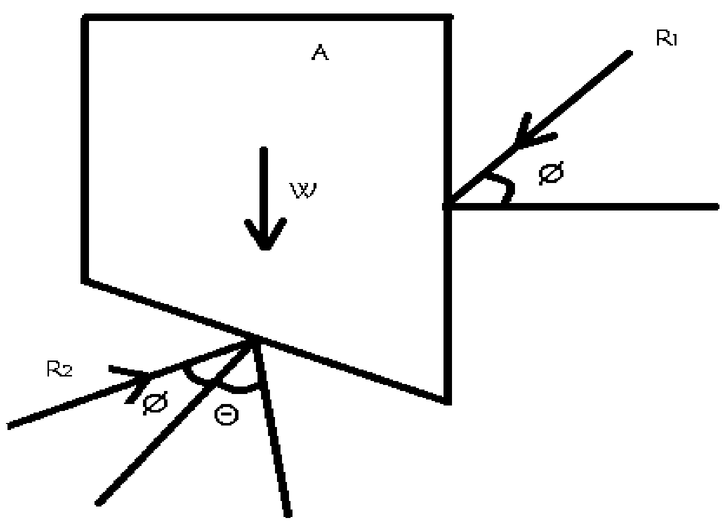
$\sum F_y = 0$

$N_2 Cos\Theta = N_3 + \mu N_2 Sin\Theta$

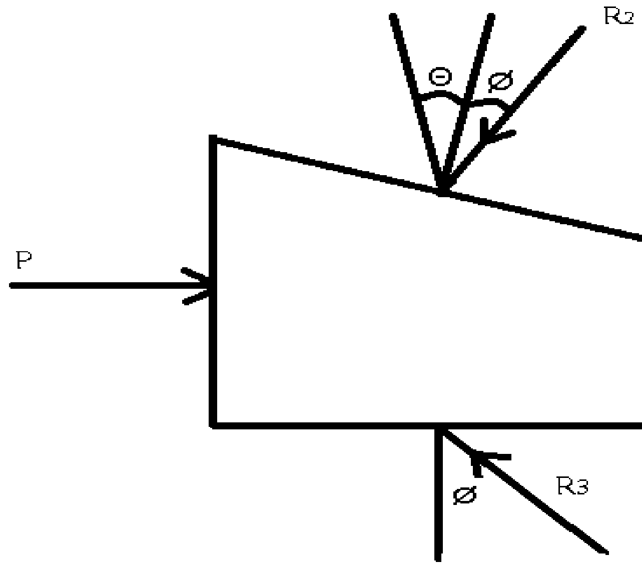
Method II:

The problem is also solved by alternate method using total reaction.

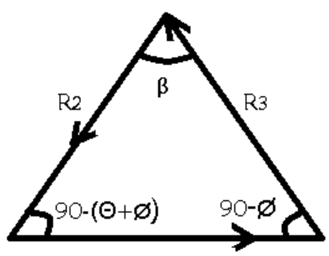
For block A:



For Wedge B:



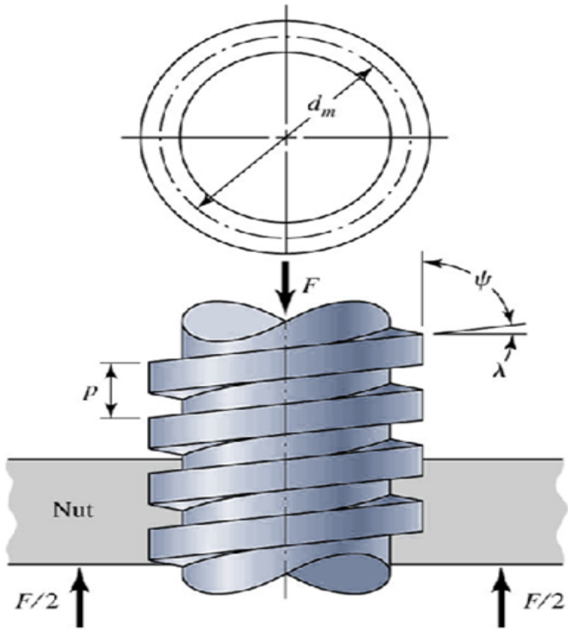
Sine rule :



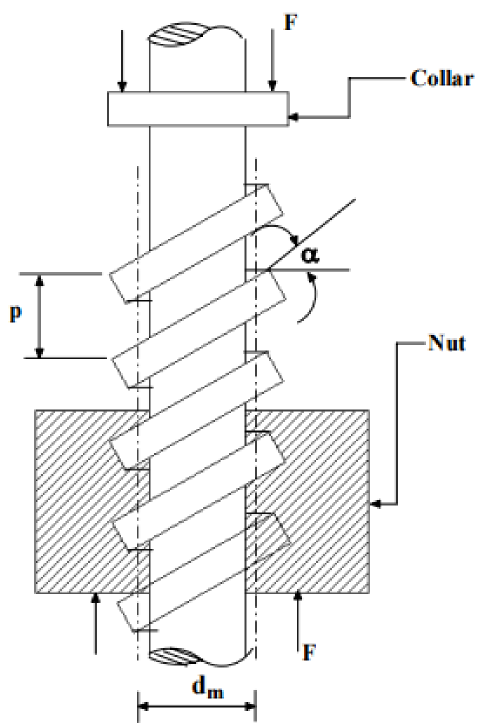
Note:

1. *When only three forces act on a free body,it is usually best to apply the Sinelaw to the triangle formed by the force polygon.*
2. *When more than three forces are involved,of which only two are unknown,it is suggested that force summations to be taken with respect to perpendicular axes,one of which coincides with one of the unknown forces.*

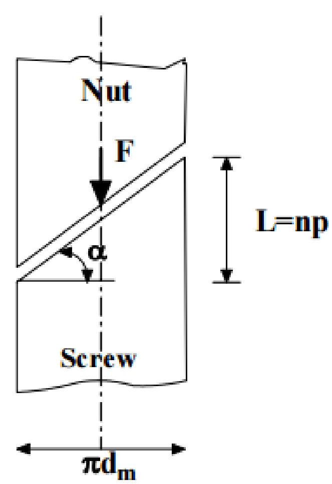
Square Threaded Screws:



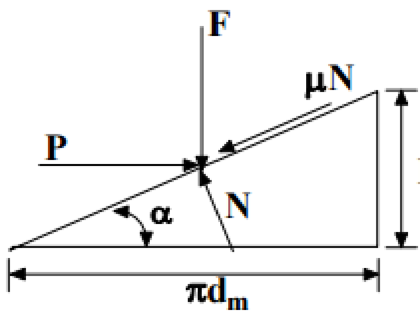
Portion of a power screw (Square)



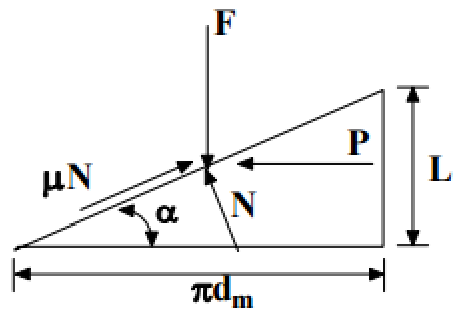
A square thread power screw



Development of a single thread



F- Forces at the contact surface for raising the load.



F- Forces at the contact surface for lowering the load.

If Q is the force applied on the arm to produce torque for lifting the body, the force P,acting in a plane perpendicular to the axis of the thread and at the mean radius of the thread.

Qa = Pr (a= arm length ; r = mean radius; a>r)

$$P = Qa / r$$

In the first case, with motion impending up the incline,

$$Q = F \tan(\phi + \alpha)$$

In the second case, with motion impending down the plane,

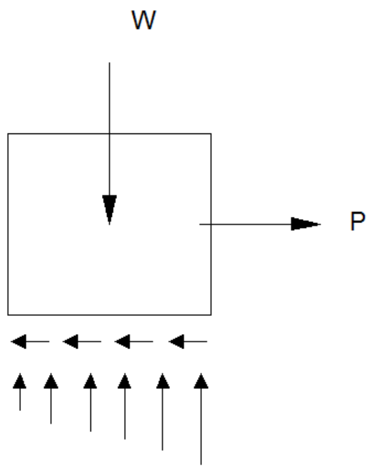
$$Q = F \tan(\phi - \alpha)$$

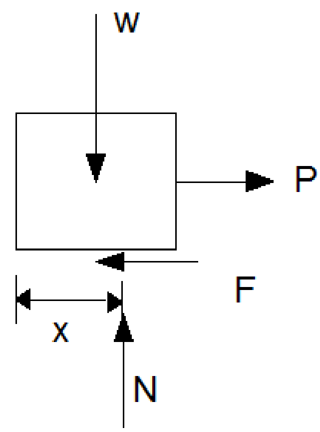
Note:

1. It is evident that if the screw is to be self-locking, the angle of friction, ϕ must be larger than the pitch angle, α.
2. In case of over hauling , $\phi < \alpha$

2. Connected Bodies

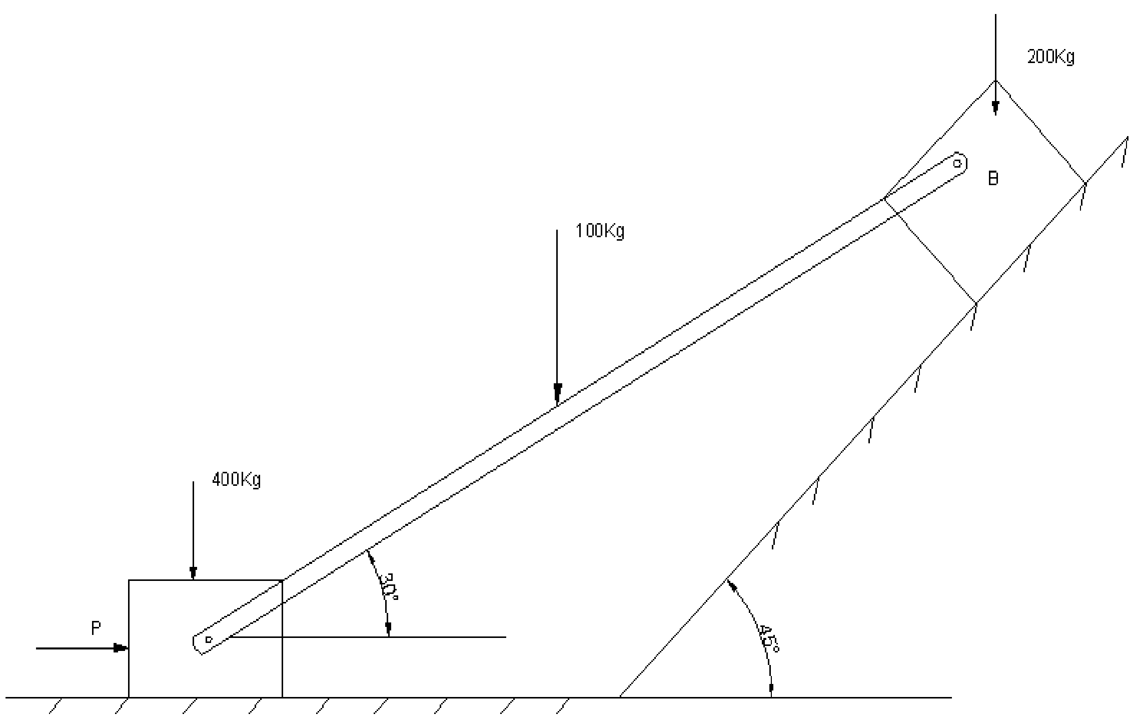
The application of a force P to the block of weight W causes the variable ground reactions and frictional components shown in figure below.These may be replaced by the resultant normal force N and frictional force F shown in figure,but the location x of the normal force can only be determined when the dimensions of the block and the position of P are specified.This concept and a method of avoiding the need to locate the normal force are discussed.

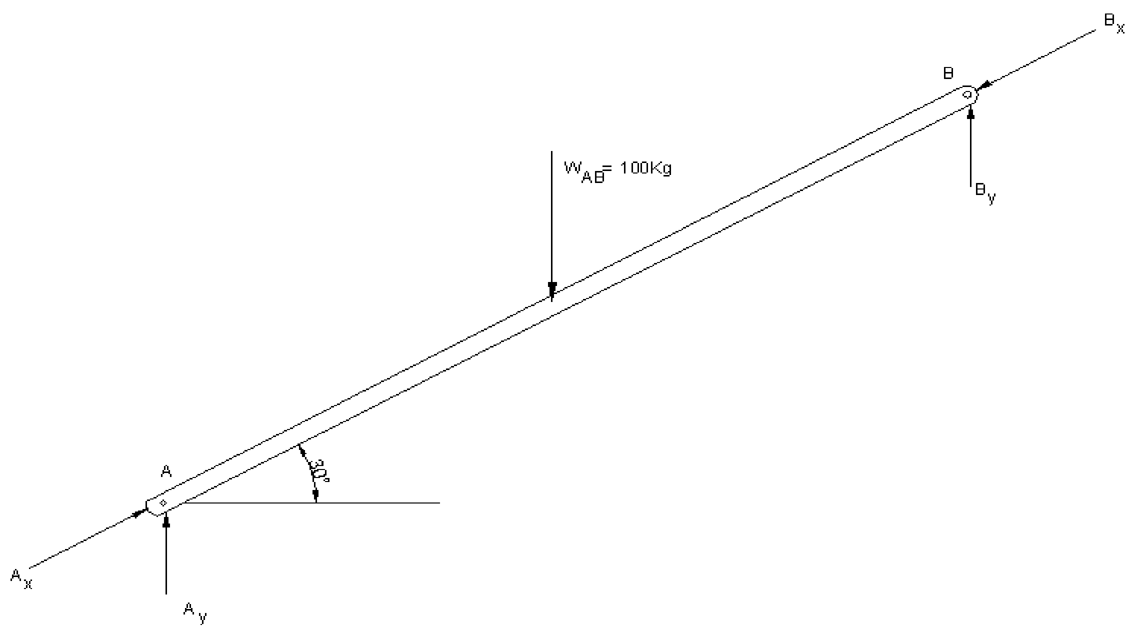




Connected bodies:

The bodies shown below are separated by a uniform strut weighing 100kg which is attached to the bodies with frictionless pins. The coefficient of friction under each body is 0.3. Determine the value of the horizontal force P that will start the system rightward.





FBD of strut:

Since the strut is a multi-force member, the directions of the pin forces at A and B are unknown. Here the components of pin forces are selected in vertical and along the strut directions. This has the advantage of permitting moment summations about A & B to eliminate three of the four unknowns and determine B_Y & A_Y directly and independently.

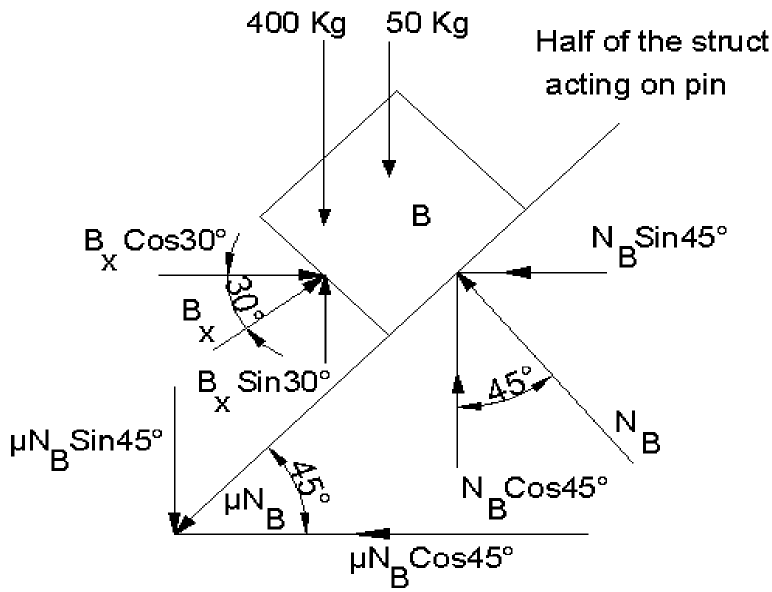
Since the weight of the strut is at its midpoint, its length is immaterial and we obtain

$B_Y = A_Y =$ kg.

So taken along the strut gives

$$B_x = A_x$$

FBD of block B:



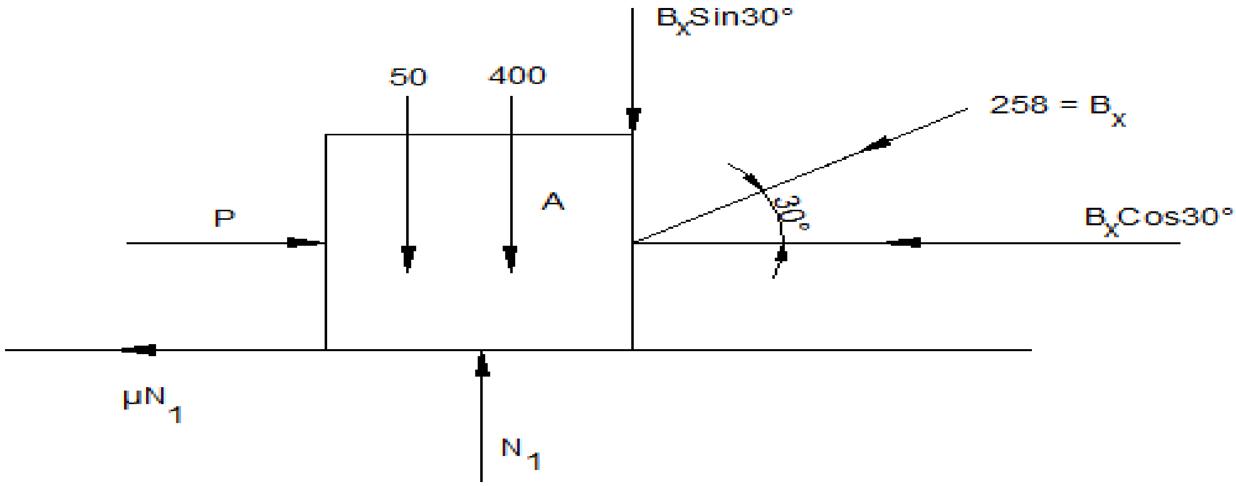
…………………….. (1)

Substituting in equation (1)

kg

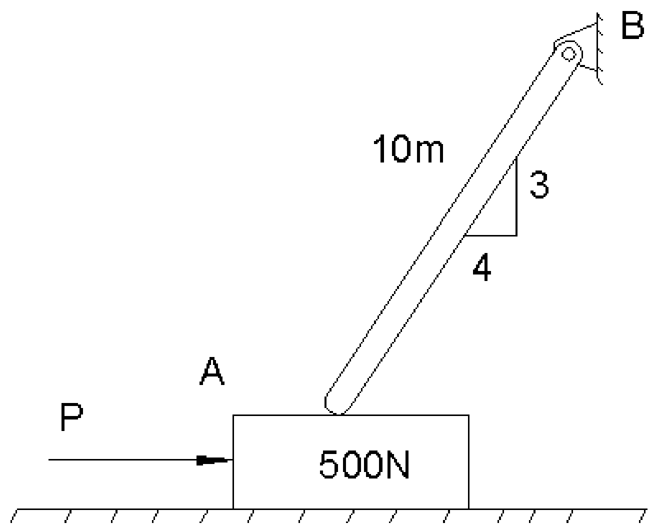
kg

FBD of block A:



But
kg
kg

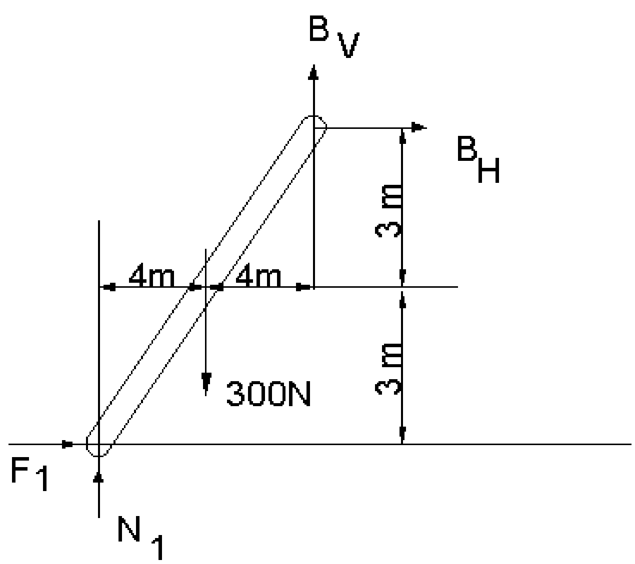
kg

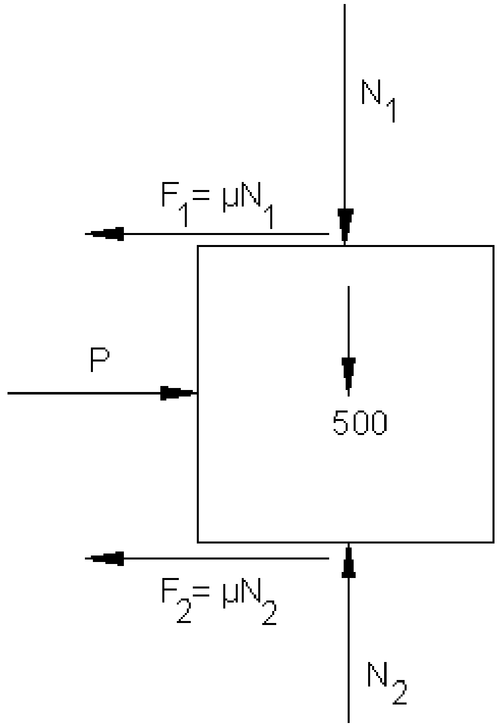


3. A uniform bar AB, 10m long and weighs 300N is hinged at B and rests upon a 500N block at A as shown in figure. If the coefficient of friction is 0.4 at all contact surfaces, find the horizontal force P required to start moving the 500N block.

Sol: The impending motion of the block is towards right, the frictional force must resist the motion hence it acts towards left on the block.

<u>FBD of the bar AB:</u>





Where

Moments about point B,

FBD of the block :

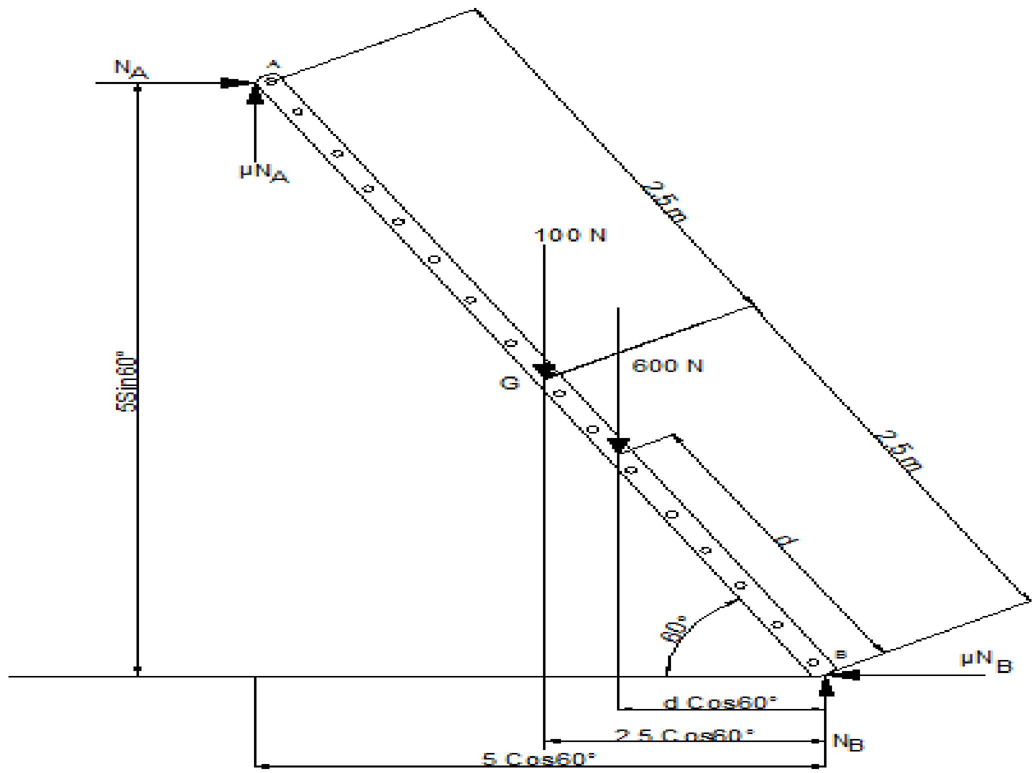
N

4. Ladder Problem

- Many a times, we come across the uses of ladder for attending the higher heights. Ladders are used by painters and carpenters who want peg a nail in the wall for mounting a frame.
- We observe that care is taken to place the ladder at appropriate angle with respect to ground and wall. We try to adjust the friction offered by the ground and wall in contact with ladder. Also sometimes we prefer to hold the ladder by a person for safety purposes.
- The forces acting on ladder are normal reactions, frictional forces between the ground, the wall and the ladder, weight of the ladder and the weight of the man climbing the ladder.
- Considering the free body diagram of ladder, we get general force system. The simplification of the system by considering equilibrium condition can be worked out by following equations:

, and

1. A uniform ladder weighing 100N and 5 meters long has lower end B resting on the ground and the upper end A resting against a vertical wall. The inclination of the ladder with horizontal is 60˚. If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance up along the ladder a man weighing 600N can ascent without causing it to slip.



Sol. Consider the FBD of the ladder

0.25

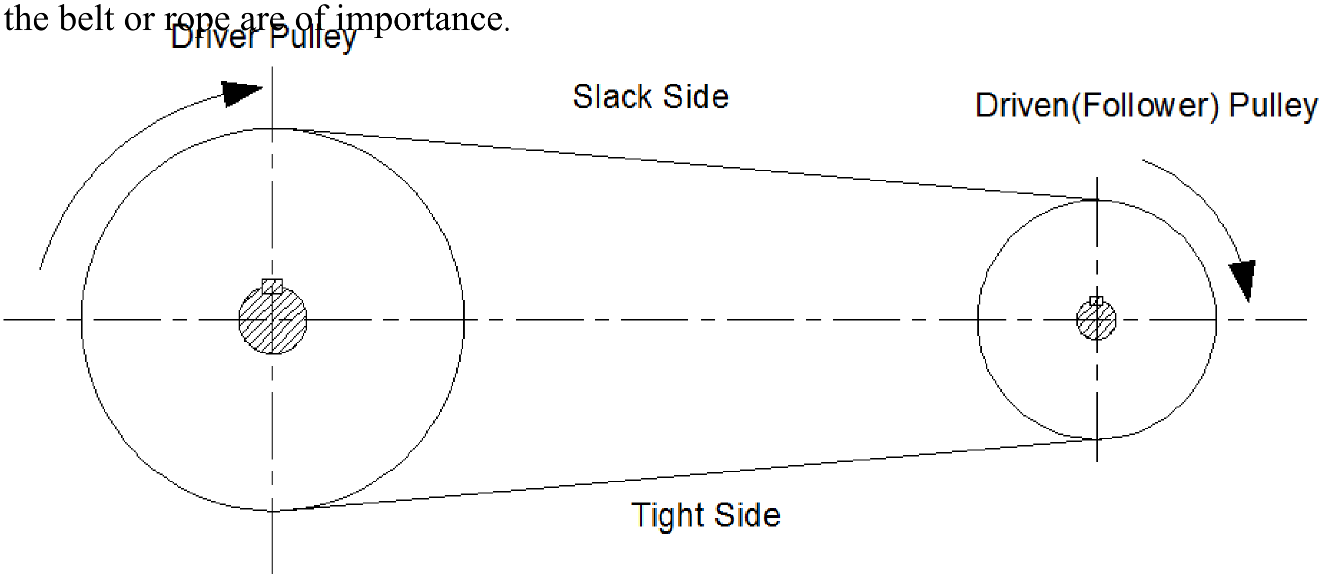
$100 \times 2.5 \cos60^\circ + 600 \times d \cos60^\circ - \times 5 \sin60^\circ - \times 5 \cos60^\circ = 0$

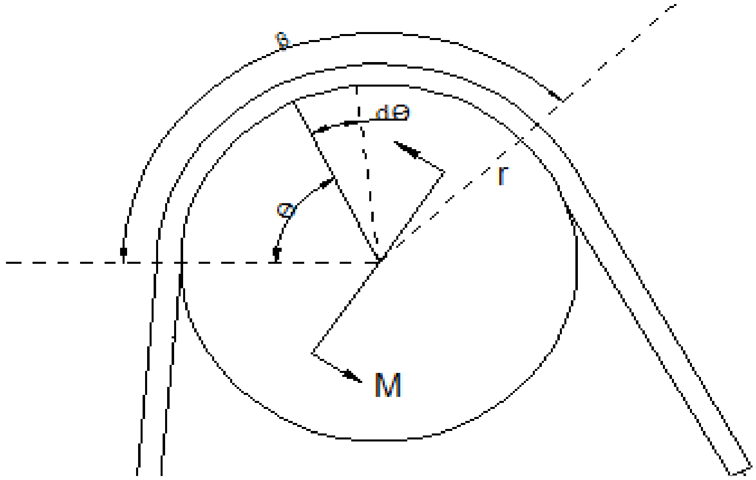
$100 \times 2.5 \cos60^\circ + 600 \times d \cos60^\circ - 164.71 \times 5 \sin60^\circ - 0.25 \times 164.71 \times 5 \cos60^\circ = 0$

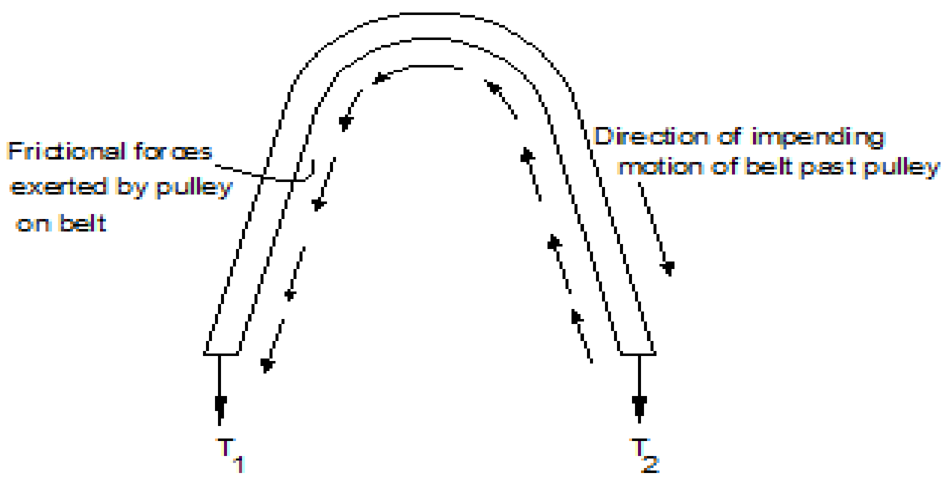
$d = 2.304$m.

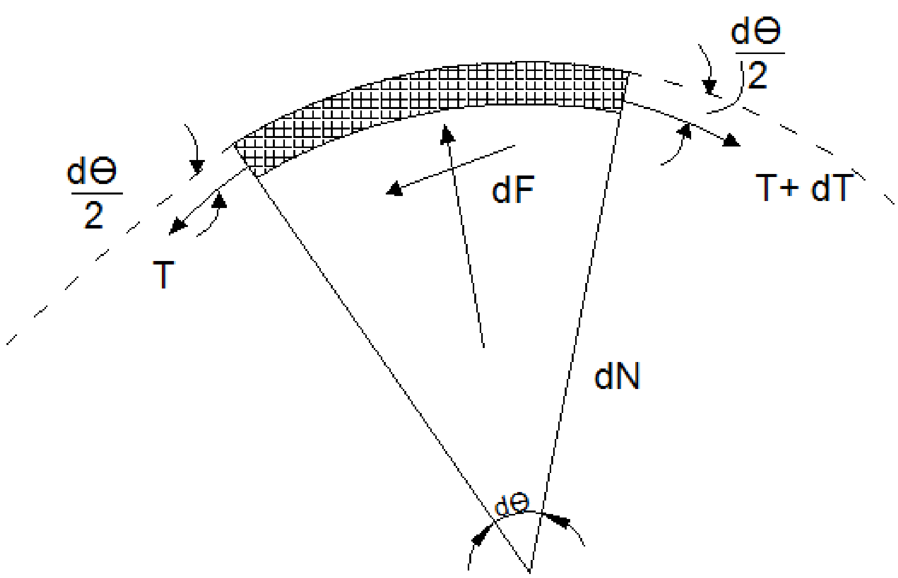
5. Belt Friction

Belt or rope is wrapped around the pulleys to transmit power or effectively used for braking systems. In order to evaluate the effectiveness of the system, the tensions in the belt or rope are of importance.









<u>Belt friction analysis:</u>

- to overcome the resistance M of the pulley to clockwise rotation.
- The friction forces cumulatively make up the difference between.
- Consider a small element, on which the belt tension increases from T at angle to (T+dT) at angle (.

These tensions are balanced by the differential normal force dN and the differential friction force .

……………………………………………………………(1) since

…………………………………… (2)
Since in radians
& neglecting term since it is negligibly small.
……………………………………………………. (3)
When slipping impends,

From equation (1) & (3)

in radians

The frictional resistance in the belt drives increases in exponential form.

Velocity ratio:

It is defined as the ratio of the velocity of the driven (follower) to the velocity of the driver.

Let N_1 = Speed of the driver in rpm

d_1 = Diameter of the driver

N_1 = Speed of the driven in rpm

D_2 = Diameter of the driven

Length of the belt passing over the two pulleys per minute will be the same

$\pi d_1 N_1 = \pi\, d_2\, N_2$

$N_1 / N_2 = d_2/d_1$

References:

1. **Ferdinand L. Singer, "Engineering Mechanics-Statics & Dynamics", Harper International Edition.**
2. **J.L.Meriam, L.G.Kraige, "Engineering Mechanics-Statics", John Wiley & Sons Pte Ltd.**
3. **Joseph F. Shelley," Vector Mechanics for Engineers". Mc Graw Hill international Editions.**