## FACULTY OF ENGINEERING

B.E. 2/4 (ECE/M/P/ AE/CSE) II - Semester (Main) Examination, June 2014

## Subject : Mathematics - IV

Time : 3 hours
Max. Marks : 75

## Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B. PART - A (25 Marks)

1 Evaluate $\int_{C} \frac{2 z+1}{(z+1)(z-1)(z+2)} d z$, where c is $|z|=\frac{3}{2}$ by using residue theorem.
2 Evaluate $\int_{0}^{1+i}\left(x^{2}+i y\right) d z$ along $y=x$.
3 Find the zeros of $f(z)=e^{z}-1$.
4 Determine the points at which the mapping $\mathrm{w}=\sin \pi \mathrm{z}$ is not conformal.
$5 \mathrm{~A}, \mathrm{~B}$ are two mutually exclusive events of a random experiment. If $P(A U B)=0.75$ and $P(\bar{A})=0.6$ then find $\mathrm{P}(\mathrm{B})$.
6 Find the expectation of Gamma variate with one parameter.
7 A continuous random variable $x$ has the pdf
$f(x)=\left\{\begin{array}{ll}a+b x & 0 \leq \mathrm{x} \leq 1 \\ 0 & \text { elsewhere }\end{array}\right.$ If the mean of the distribution is $\frac{1}{3}$, find the values of $\mathrm{a}, \mathrm{b}$.
8 Write 4 properties of normal distribution.
9 Write the normal equations of the curve $y=a+b x+c x^{2}$.
10 Two random variables have the regression lines with equations $3 x+2 y=26$ and $6 x+y$ $=31$. Find the mean values and the correlation coefficient between $x$ and $y$.

PART - B (50 Marks)
11 a) Show that $J\left(\frac{u, v}{x, y}\right)=\left|f^{\prime}(z)\right|^{2}$ where $\mathrm{f}(\mathrm{z})=\mathrm{u}(\mathrm{x}, \mathrm{y})+\mathrm{iv}(\mathrm{x}, \mathrm{y})$, and J is the Jabocian.
b) State and prove Cauchy's integral formula.

12 a) Evaluate $\int_{-\infty}^{\infty} \frac{d x}{\left(x^{2}+a^{2}\right)^{2}}$.
b) Expand $\frac{7 z-2}{(z+1) z(z-2)}$ in the region $1<|z+1|<3$

13 a) State and prove Baye's theorem.
b) If $X$ is a random variable with probability distribution function

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $P(x)$ | 0.15 | 0.1 | 0.05 | 0.3 | 0.2 | 0.1 | 0 |

find $E(x+1), E(3 x+2), V(3 x+4)$.
14 a) Derive the MGF of normal distribution.
b) Two random samples of sizes $9 \& 6$ are given the following values of the variate.

| Sample-I | 15 | 22 | 28 | 26 | 18 | 17 | 29 | 21 | 24 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample-II | 8 | 12 | 9 | 16 | 15 | 10 | - | - | - |

Test the difference of estimates of the population variances at $5 \%$ level of significance. [ $\mathrm{F}_{0.05}$ at $(8,5) \mathrm{df}=3.69$ ].

15 Find the correlation coefficient between $x \& y$ for the following values and also the regression lines.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 10 | 12 | 16 | 28 | 25 | 36 | 41 | 49 | 40 | 50 |

16 a) Prove that $u=x^{2}-y^{2}, v=\frac{y}{x^{2}+y^{2}}$ are harmonic.
b) Find the bilinear transformation which maps the points $z=-1, i, 1$ into $w=1,-i,-1$.

17 a) Fit a Poisson distribution to the following data.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 46 | 38 | 22 | 9 | 1 |

b) Evaluate $\int_{0}^{2 \pi} \frac{d \theta}{2+\sin \theta}$.

