Unit-I: Part-4: Conic sections (Ellipse, parabola, hyperbola and rectangular hyperbola)

Theory Questions

1. What is a conic?

A: It is a locus of point moving in a plane in such a way that the ratio of its distances from a fixed point (focus) and a fixed line (directrix) is always constant.

The fixed point is called as focus and the fixed line is called as directrix.

2. In a conic, the line passing through the fixed point & perpendicular to the fixed line is called the _____ (Ans: Axis).
3. The point at which the conic cuts its axis is called as _____ (Ans: Vertex)

4. Define eccentricity.

A: Eccentricity is the ratio of distance of the point from the focus to the distance of the point from directrix. (e = PF/PD)

5. State the values of eccentricity for different conics.

A: Ellipse: e<1; Parabola: e = 1; Hyperbola: e>1; rectangular hyperbola: e = √2.

6. Explain how a cone is to be cut to get various conic sections with simple sketches.

A: When the section plane is inclined to the axis and cuts all the generators on one side of apex, the section (true shape of cut portion) is an ellipse.

When the section plane is inclined to the axis and is parallel to one of the generators, the section is a parabola.

When the section plane cuts both the parts of the double cone on one side of the axis, the section is a hyperbola. (refer to the figure from text book in introduction of conic sections)

7. Explain the oblong method of drawing an ellipse. (Refer to the construction procedure).

8. The locus of a point P moving in such a way that the sum of its distance from two fixed points is always constant is called as _____. (ellipse; as $PF_1+PF_2 = c = 2a$)

9. The locus of a point P moving in such a way that the difference between its distances from two fixed points is always constant is called as _____. (Hyperbola; $PF_1-PF_2 = c$)

Problems

I: Problems on general method (foci-eccentricity method) (common to all the 3 curves)

10. A **fixed point** is 50 mm away from a **fixed line**. Draw the path traced by a point P moving such that its distance from the **fixed line** is
    (i) **3/2** times its distance from the fixed point. (e = PF/PD = 2/3; ellipse)
    (ii) **equal** to its distance from the fixed point. (e = PF/PD = 1; parabola)
    (iii) **2/3** times its distance from the fixed point. (e = PF/PD = 3/2; hyperbola)

    Also draw **tangent** and **normal** to the curve at a point 65 mm from the focus.
    (refer to construction)

11. A **fixed point** is 50 mm away from a **fixed line**. Draw the path traced by a point P moving such that its distance from the **fixed point** is
(i) **3/2** times its distance from the fixed line. (e = PF/PD = 3/2; hyperbola)
(ii) **equal** to its distance from the fixed line. (e = PF/PD = 1; parabola)
(iii) **2/3** times its distance from the fixed line. (e = PF/PD = 2/3; ellipse)

    Also draw **tangent** and **normal** to the curve at a point 50 mm from the directrix.
(refer to construction)

    Note: In problems 10 ad 11, data is similar but the way in which eccentricity is defined is to be observed.

12. The vertex of a hyperbola is 65 mm from its focus. Draw the curve if the eccentricity is 3/2, Draw a normal and tangent at a point on the curve, 75 mm from the directrix. (refer to construction of hyperbola by general method)

II: *Construction of Ellipse:*
(Oblong method, concentric circles method and arc of circles method)

13. Construct an ellipse whose major axis is 150 mm & minor axis is 100 mm long by
(i) Oblong method (ii) Concentric circles method. (iii) Arc of circle method.

Draw a tangent and normal to the ellipse at a point on it 40 mm above the major axis.

14. Two **fixed points** A and B are 100 mm apart. Trace the complete path of a point P moving (in the same plane as that of A and B) in such a way that **sum of its distances** from the fixed points A and B is always constant and is equal to 125 mm. Name the curve.
(hint: Since PA + PB = c = 2a is similar to $PF_1 + PF_2 = c = 2a$; it is ellipse with distance between foci(AB or F_1F_2) = 100 and length of major axis = 125) (any method)

15. Draw an ellipse whose major axis is 100 mm & minor axis is 60 mm. Locate its foci & draw a tangent & normal to the curve at 40 mm from the center of the ellipse. (any method)

16. Draw & determine the length of minor axis of an ellipse with major axis of 140 mm & the distance between foci of 96 mm using concentric circles method.

17. The foci of an ellipse are 90 mm apart and the minor axis is 65 mm long. Find the length of the major axis and draw half the ellipse by concentric circles method and half ellipse by oblong method.

18. Inscribe an ellipse in a parallelogram of 150 mm & 100 mm long and an included angle of 120^0. (parallelogram method-similar to oblong method)

19. Two points A and B are 100 mm apart. A point C is 75 mm from A and 60 mm from B. Draw an ellipse passing through A, B and C. (ellipse by parallelogram method)

II: *Construction of Parabola:*
(Oblong method, tangent method or triangle method)

20. **A ball** is thrown from a ground, travels a maximum **horizontal distance** of **8.5 meters** & reaches a **maximum height** of 5.0 meters.Trace the path of the ball, assuming it to be parabolic (refer construction by oblong or rectangle method)

21. Draw 2 parabolas inside a rectangle of 125 mm X 85 mm such that their axes bisect each other. (draw rectangle of 125x85 & draw the parabolas along x and y direction by oblong method).

22. A fountain jet discharges water at an angle of 45^0 to the horizontal. It travels a maximum horizontal distance of **8.5 meters** & falls on the ground. Trace the path of the jet, assuming it to be parabolic. (refer tangent method for parabola)

23. Draw a parabola passing through three vertices of a triangle of sides 30 mm, 45 mm and 60 mm. The comer of the triangle common to 45 mm and 60 mm sides lies on the axis of parabola. Use any method.

III: Construction of Hyperbola and Rectangular Hyperbola:

(Arc of circles method for pair of hyperbolas and Asymptotes method for Rect.Hyperbola)

24. Two **fixed points** are 50 mm apart. Draw the locus of a point P moving in such a way that **difference of its distances** from the fixed points is always constant and is equal to 20 mm.
(as PF_1-PF_2 = c, it is a hyperbola, to be drawn by arc of circles method similar to ellipse)

25. A point P is 40 mm and 50 mm away from two straight lines OA and OB which are at right angles to each other. Draw a rectangular hyperbola through P, with OA and OB as asymptotes, showing at least 8 points. (refer to the asymptotes method for rectangular hyperbola)

26. Two straight lines OA and OB make an angle of 75^0 between them. Point P 40 mm from OA and 50 mm from OB. Draw a hyperbola through P within 10 mm distance of each line. (similar to above problem, leave 10 mm away from OA and OB).

For all the problems mentioned above, refer to the solutions enclosed. Basic methods have been discussed for all the above curves.

For some problems, solutions have not been given and they are to be solved as assignment problems.