

Overview of Conic Sections by General method

While preparing for conic sections, the following steps are to be followed:

- 1) What is the distance between the focus and directrix?
(A) This distance is taken as DF.
- 2) What is the eccentricity of the given curve?
(A) To calculate eccentricity, we use the definition of eccentricity

$$e = PF/PD$$

PF is the distance of the point P from Focus (fixed point)
PD is the distance of the point P from the directrix (fixed line).

Note: In the problems, the ratio of PF/PD has to be found.

This may be given in a statement form in terms of

- (i) ratio of distances between **fixed point** and **fixed line (PF/PD)**
- (ii) ratio of distances between **fixed line** and **fixed point (PD/PF)**.

For all conic sections, we have to remember that $e = PF/PD$

And hence using the statement of problem we can find e.

If $e < 1$	—————→	<i>the curve is Ellipse</i>
$e = 1$	—————→	<i>the curve is Parabola</i>
$e > 1$	—————→	<i>the curve is Hyperbola.</i>

E.g.:

- 1) A point P moves such that its **distance from fixed point** is **2/3** times its **distance from the fixed line**. **Trace the path of point P** when **fixed point** is 50 mm away from **fixed line**.

(A) Here, **DF = 50 mm**; Relation is **PF = 2/3 PD** and hence $e = PF/PD = 2/3$ ($e < 1$);
Curve is **Ellipse**.

- 2) A point P moves such that its ratio of its distance from **fixed line** to its distance from the **fixed point** is **2/3**. Trace the path of point P when fixed point is 50 mm away from fixed line.

(A) Here, **DF = 50 mm**; Relation is **PD = 2/3 PF** and hence $e = PF/PD = 3/2$ ($e > 1$);
Curve is **Hyperbola**.

Steps of Construction by the General method are as follows:

- 1) Draw a straight vertical line AB of any length (directrix).
- 2) Draw a horizontal line DC perpendicular to AB (Axis) at any point.
- 3) From D, mark F (Focus) at given distance from AB (Directrix).
- 4) Take $e = \frac{PF}{PD}$ ($e = m/n$) and hence divide DF into $(m+n)$ no. of equal parts.

E.g. If $e = 2/3$, then divide DF into $(2+3) = 5$ equal parts.

- 5) Mark V (Vertex) at m^{th} part after F.
(e.g.: if $e = 2/3$, then V is 2^{nd} part after F).
- 6) Draw $VE \perp VF$ such that $VE = VF$.
- 7) Join DE and extend it.
- 8) On the axis, mark a no. of points after F at 10 mm each and label them as 1, 2, 3, etc.
- 9) On 1, 2, 3, etc draw vertical lines to cut the Line DE extended at $1'$, $2'$, $3'$, so on.
- 10) To get points of the curve, we need to draw arcs.
- 11) For all arcs, centre is F (focus); Radius is $1-1'$, $2-2'$, $3-3'$, etc.
- 12) With F as centre and radius = $1-1'$, cut arc on line $1-1'$. Similarly $2-2'$, $3-3'$ etc cut arcs on lines $2-2'$, $3-3'$, etc and label the points as P_1 , P_1' , etc above and below the axis.
- 13) Join all these points to get the required conic section.

Tangent and Normal to the conic sections:

- 1) Mark the point M where we want to draw tangent and normal either from the directrix or from the focus.
- 2) Join MF and at F, draw a line \perp to MF to cut the directrix at T. T is the starting point of the tangent. Join TM & extend to get Tangent TT' .
- 3) Draw the normal NN' \perp to the tangent TT' at M.