## Unit-I-Part-2: Involutes

## Theory Questions

1. Define an involute.

A: Involute is defined as the curve traced by the end of a thread as it is unwound around a line, polygon or a circle, the thread being kept tight.

Involute is also defined as the curve traced out by a point on a straight line when the line rolls along a circle or a polygon without slipping.

2. Differentiate between a cycloid and an involute

A: A cycloid is a curve in which the circle rolls along a straight line where as an involute is a curve in which the line rolls along the circle or a polygon.
3. What are the applications of involutes and cycloids?

A: Involutes shapes are used as teeth profiles in gears as they give less noise, vibrations, wear and tear. The involute of a circle is also an important shape in gas compressing, as a scroll compressor can be built based on this shape. Scroll compressors make less sound than conventional compressors, and have proven to be quite efficient.
4. What are the applications of cycloidal curves?

A: Cycloidal curves are used in the design of tooth profiles of gears. It is also used in the design of conveyor of mould boxes in foundry shops. Cycloidal curves are also commonly used in kinematics (motion studies) and in mechanisms that work with rolling contact.

## Problems

Common procedure for involutes of lines and polygons:
Step 1. Draw the required line or polygon of given dimensions and give labelling as A, B, etc starting from the left bottom corner for standardisation.


Step 2: Draw extension lines on each labelled point in the opposite direction of the next labelling sequence. $\mathrm{E} . g$. On AB , draw extension line in direction of BA , on BC , draw extension line in direction of $C B$; for $C D$, draw along $D C$, etc. This sequence should be followed to have uniformity and avoid confusion.


Step 3. Starting from point A, draw arcs of radius $=a, 2 a, 3 a, 4 a$, etc where $a$ is the length of the line or side of the polygon such that the arcs end on each extensions.


Using the steps mentioned above, solve the following problems.
5. Draw an involute of a line of 20 mm for (i) 1 convolution; (ii) 2 convolutions.
A. One convolution for a line means 2 arcs as there are two ends, A and B. R20 and R40 will the radii for completer one convolution. Two convolutions means the involute has to be repeated again from $A$ and $B$. Length of arcs will be $R 60$ and $R 80$.
4 convolutions means 4 times involutes have to be drawn. (A,B;A,B;A,B;A,B)

6. Draw an involute of a triangle, square, pentagon and hexagon taking side as 30 mm .


The above figure is the involute of a triangle of sides 30 mm . Similarly, involutes for square, pentagon and hexagon may also be drawn.
7. Draw an involute of a circle of 40 mm diameter. Also draw a tangent and normal to the involute at 100 mm from the centre of the circle.
A. Steps: (a) To draw the involute of the circle:
(i) Draw a circle of radius 20 mm , divide it into 8 equal parts ( $45^{\circ}$ ) and label them 1, 2, 3, etc. Take the bottom most point as P and ensure that labelling is in the anticlockwise direction. P will coincide with 8.

(ii) On P, draw a horizontal line PA (tangent) of length $=2 \pi R$ or $\pi D$ (circumference) and divide this line also into 8 equal parts. Here $L=\pi * 40=125.6 \mathrm{~mm}$; Use line division. Label the parts of line as $1^{\prime}, 2^{\prime}, 3^{\prime}$, etc.

(iii) At points 1, 2, 3, 4, etc on the circle, draw tangents by keeping the drafter perpendicular to C1, C2, C3, etc.

(iv) To get the points of the involute, use the principle, $\mathrm{P}-1^{\prime}=1-\mathrm{P}_{1} ; \mathrm{P}-2^{\prime}=2-\mathrm{P}_{2}$, etc. Keep 1 as center, radius $=P-1$ ', cut arc on line 1 to get $P_{1}$.
Similarly with 2 as center, radius $=P-2^{\prime}$, cut arc on line 2 to get $P_{2}$. Thus using this principle, cut the other arcs on the tangents to get points $P_{3}, P_{4}$, etc. Last point is PA and need not mark.
(v) Join all the points $P_{1}, P_{2}, P_{3}, \ldots A$ to get the required involute.

(b) To draw tangent and normal to the involute at 100 mm from the centre of the circle: Steps:
(i) With C as center and radius 100 mm , mark point N on the involute.
(ii) On CN, locate midpoint O and draw a semicircle on CN with OC or ON as radius.
(iii) The point where the semicircle cuts the circle is the point $M$ (starting point of normal) and draw the normal by joining NM.
(iv) Draw the tangent TT' (or ST) perpendicular to NM.


The above figure is for 12 points but 8 parts are sufficient if time is a constraint.

## Assignment problem:

Draw a path of string when it is wound on a circle of 40 mm diameter without slipping. The length of the string is 150 cm long. Name the curve \& write its practical applications. A. When the length of string is $>\pi \mathrm{D}$, the involute will extend further into the circle. Refer Article 6.3 in ND Bhat textbook for the solution to this problem.

